

## Exponentials and Logarithms Essential Practice



**Skill:** Reduction to Linear Form

### Questions

Attempt these questions independently showing full and clear solutions. Check each answer as you go.

1. Reduce each of the following equations in variables  $x$  and  $y$  into linear form using natural logarithms, stating the variables that must be plotted against each other in order to achieve a straight line. State also the gradient and y-intercept of the resulting straight line.

a.  $y = ax^b$

b.  $y = ab^x$

c.  $y = ae^{bx}$

d.  $y = a \times 2^{bx}$

e.  $y = 6 \times 7^x$

f.  $y = ax^{3b}$

2. At time  $t$  mins the temperature  $\theta^\circ\text{C}$  of a liquid is modelled by the equation

$$\theta = ab^{-t}$$

where  $a$  and  $b$  are unknown constants.

- Show that  $\log_{10} \theta$  when plotted against  $t$  will give a straight line, and state the y-intercept and gradient of this line in terms of  $a$  and  $b$ , respectively.
- The straight line obtained in (i) passes through the points  $(2, 1.05)$  and  $(5, -0.03)$ . Determine the values of  $a$  and  $b$  correct to 2 d.p.
- Hence use the model to predict:
  - the initial temperature of the liquid
  - the temperature of the liquid after 2.5 minutes
  - the temperature of the liquid in the long term
  - the time at which the liquid will have temperature  $40^\circ\text{C}$

3. The population  $P$  (measured in thousands of people) in year  $t$  (measured in number of years after 1970) of the island of Sodor is shown in the table below.

Year	1970	1980	1990	2000	2010
$t$	0	10	20	30	40
$P$	15.0	19.1	24.6	31.6	40.3

It is believed that  $P$  and  $t$  are related by the formula  $P = a \times 10^{kt}$  where  $a$  and  $k$  are constants to be determined.

[Question continues on next page]

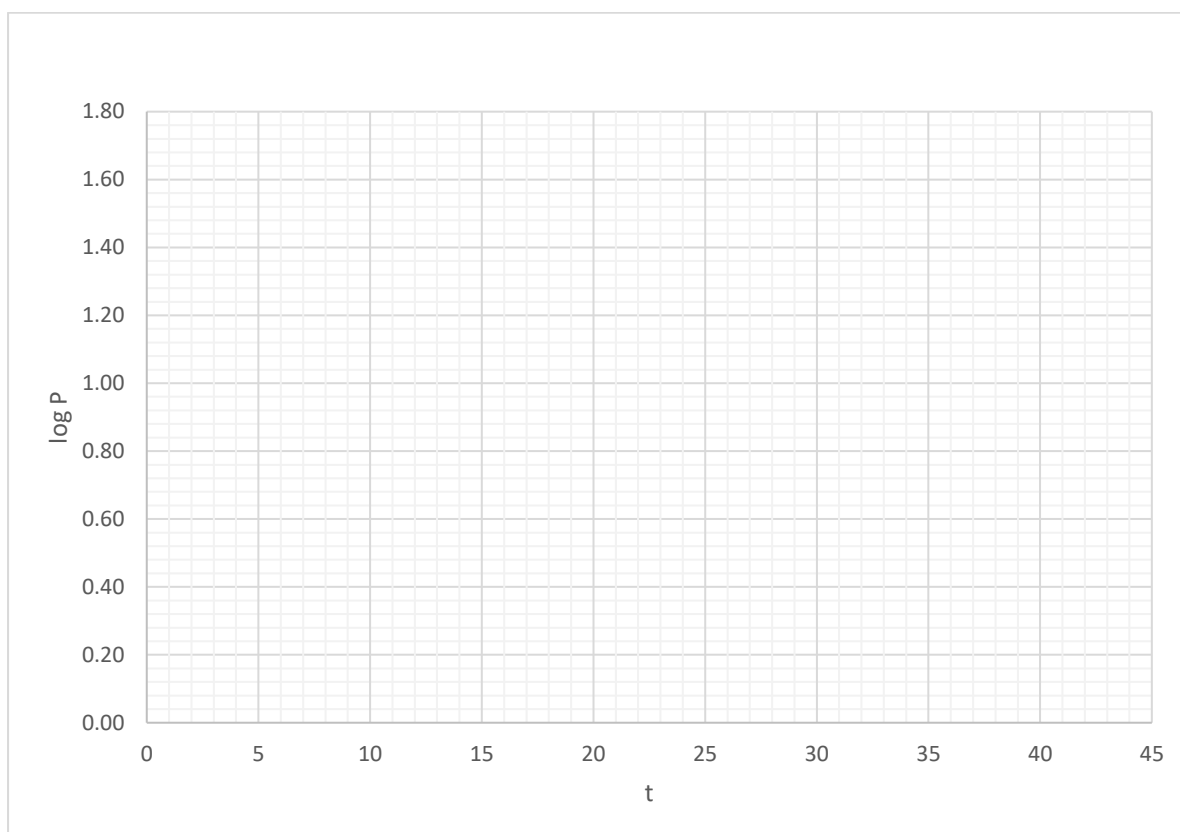


a. Show that if we assume the above equation holds true, then a straight line should be obtained when  $\log_{10} P$  is plotted against  $t$ . State the gradient and y-intercepts of this line in terms of the unknown constants.

b. Complete the table below to obtain values of  $\log_{10} P$  to 3 s.f.

$t$	0	10	20	30	40
$\log_{10} P$					

c. Use the table from (b) to plot the graph of  $\log_{10} P$  against  $t$  on the axes below, drawing a line of best fit by eye.



d. Use the relevant features of your graph to estimate the values of the constants  $a$  and  $k$  to 3 s.f.

e. Hence write an equation for  $P$  in terms of  $t$ .

f. Use your equation in part (e) to predict:

i. the population of Sodor in 1985.

ii. the year in which the population reaches 35 000.

g. Explain why this model might not be realistic in the long term.