Skill: Reduction to Linear Form

Solutions

1.

1a) $y = ax^b \Rightarrow \ln y = \ln(ax^b) \Rightarrow \ln y = \ln a + \ln x^b$

$\Rightarrow \ln y = \ln a + b\ln x$

$Y = c + mx$

$\therefore$ when $\ln y$ is plotted against $\ln x$, a straight line of gradient b and y-intercept $\ln a$ is obtained

b) $y = ab^x \Rightarrow \ln y = \ln(ab^x) \Rightarrow \ln y = \ln a + \ln b^x$

$\Rightarrow \ln y = \ln a + x\ln b$

$Y = c + Xm$

$\therefore$ when $\ln y$ is plotted against x, a straight line of gradient $\ln b$ and y-intercept $\ln a$ is obtained

c) $y = ae^{bx} \Rightarrow \ln y = \ln(ae^{bx}) \Rightarrow \ln y = \ln a + \ln e^{bx}$

$\Rightarrow \ln y = \ln a + bx$

$Y = c + mX$

$\therefore$ when $\ln y$ is plotted against x, a straight line of gradient b and y-intercept $\ln a$ is obtained.

d) $y = a \times 2^{bx} \Rightarrow \ln y = \ln(a \times 2^{bx}) \Rightarrow \ln y = \ln a + \ln(2^{bx})$

$\Rightarrow \ln y = \ln a + bx\ln 2 \Rightarrow \ln y = \ln a + (b\ln 2)x$

$Y = c + m\,X$

$\therefore$ If $\ln y$ is plotted against x, a straight line of gradient $b\ln 2$ and y-intercept $\ln a$ is obtained.

e/ $y = 6 \times 7^x \Rightarrow \ln y = \ln(6 \times 7^x) \Rightarrow \ln y = \ln 6 + \ln 7^x$

$$\Rightarrow \underset{Y}{\ln y} = \underset{c}{\ln 6} + \underset{X}{x}\,\underset{m}{\ln 7}$$

$\therefore$ If $\ln y$ is plotted against x, a straight line of gradient $\ln 7$ and y-intercept $\ln 6$ is obtained

f/ $y = ax^{36} \Rightarrow \ln y = \ln(ax^{36}) \Rightarrow \ln y = \ln a + \ln x^{36}$

$$\Rightarrow \underset{Y}{\ln y} = \underset{c}{\ln a} + \underset{m}{36}\,\underset{X}{\ln x}$$

$\therefore$ When $\ln y$ is plotted against $\ln x$, a straight line of gradient 36 and y-intercept $\ln a$ is obtained.

2.

2i/ $\theta = ab^{-t} \Rightarrow \log\theta = \log(ab^{-t}) \Rightarrow \log\theta = \log a + \log b^{-t}$

$$\Rightarrow \log\theta = \log a - t\log b$$

$$\underset{Y}{\log\theta} = \underset{m}{(-\log b)}\,\underset{x}{t} + \underset{c}{\log a}$$

$\therefore$ When $\log_{10}\theta$ is plotted against t, a straight line of gradient $-\log_{10} b$ and y-intercept $\log_{10} a$ is obtained.

ii/ Finding the equation of the line, we get

$$m = \frac{-0.03 - 1.05}{5 - 2} = -0.36 \quad (= -\log b)$$

$\therefore Y - 1.05 = -0.36(X - 2)$ (where $Y = \log\theta$ and $X = t$)

$\Rightarrow Y - 1.05 = -0.36X + 0.72$

$\Rightarrow Y = -0.36X + 1.77$

so $m = -\log b = -0.36 \Rightarrow b = 10^{0.36} \approx 2.29$

$c = \log a = 1.77 \Rightarrow a = 10^{1.77} \approx 58.88$

iii/ $\theta = 58.88 \times 2.29^{-t}$

a/ $t = 0 \Rightarrow \theta = 58.88 \times 2.29^{0} = 58.88°C$

b/ $t = 2.5 \Rightarrow \theta = 58.88 \times 2.29^{-2.5} = 7.42°C$

c/ Let $t \to \infty \Rightarrow 2.29^{-t} \to 0 \Rightarrow \theta \to 0$

$\therefore$ The long term temperature of the liquid is $0°C$

d/ $\theta = 40 \Rightarrow 40 = 58.88 \times 2.29^{-t}$

$\Rightarrow 0.67935 = 2.29^{-t}$

$\Rightarrow \log_{2.29}(0.67935) = -t \Rightarrow t = 0.4666$ mins

3a/ $P = a \times 10^{kt} \Rightarrow \log P = \log(a \times 10^{kt})$

$\Rightarrow \log P = \log a + \log(10^{kt})$

$\Rightarrow \log P = \log a + kt$

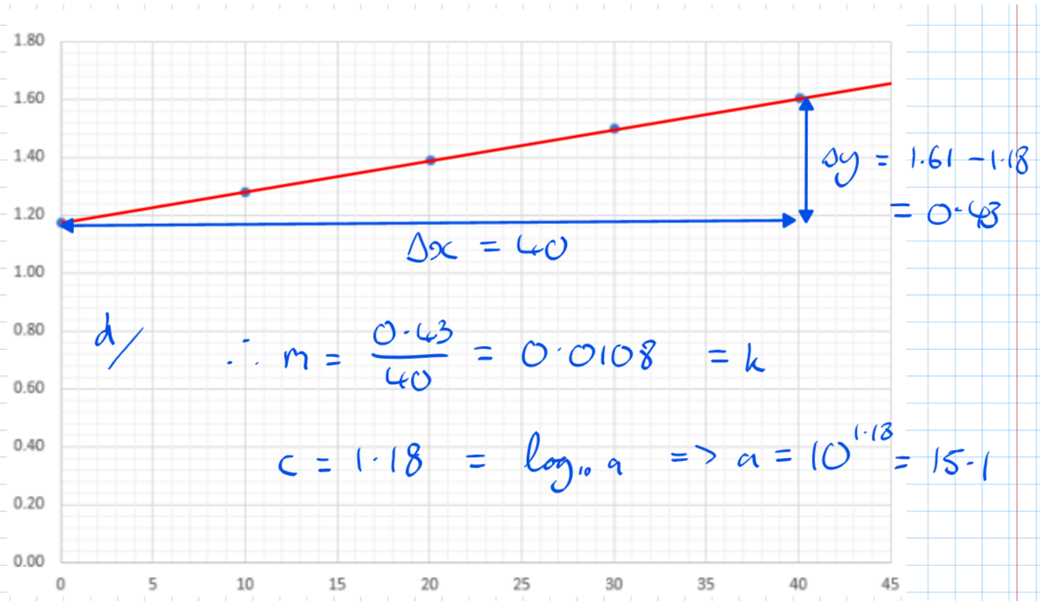
$Y = c + mx$

$\therefore$ If $\log_{10} P$ is plotted against t, a straight line with gradient k and y-intercept $\log_{10} a$ is obtained.

b/

Year	1970	1980	1990	2000	2010
t	0	10	20	30	40
$\log_{10} P$	1.18	1.28	1.39	1.50	1.61

c/



d/ $\therefore m = \dfrac{0.43}{40} = 0.0108 = k$

$c = 1.18 = \log_{10} a \Rightarrow a = 10^{1.18} = 15.1$

e/ $P = 15.1 \times 10^{0.0108t}$

f/ i/ Let $t = 15 \Rightarrow P = 15.1 \times 10^{0.0108(15)} = 21.9$

ii/ Let $P = 35 \Rightarrow 35 = 15.1 \times 10^{0.0108t}$

$\Rightarrow 2.3179 = 10^{0.0108t}$

$\Rightarrow \log_{10}(2.3179) = 0.0108t$

$\Rightarrow t = \dfrac{\log_{10}(2.3179)}{0.0108} = 33.8$

$\therefore 1970 + 33 = 2003$

g/ The population, according to the model, will continue to rise exponentially which is unrealistic due to the limited space on the island.