Exponentials and Logarithms Worked Examples

Skill:	Reduction to Linear Form

Objective: Rearranging a nonlinear equation into the form $y = mx + c$

Key Teaching Guidance

- *Take students through rearranging a nonlinear equation into linear form using logarithms.*
- *Be very clear with students that the first thing they need to do is identify the variables in the given equation and distinguish these from the constant algebraic terms.*
- *When the equation is in linear form, I suggest you write $Y = mX + c$ (Y and X are capitals here as it is likely the lowercase letters x and y will be used in the question) and show terms in this line equation being mapped to the variables in your transformed equation.*

Worked Example 1

Rearrange the following to the form $Y = mX + c$ where Y and X are functions of the coordinate variables y and x, respectively, using natural logarithms. State the values of the gradient and the y-intercept in each case.

a. $y = 3x^2$

$\ln y = \ln(3x^2) \Rightarrow \ln y = \ln 3 + \ln(x^2)$

$\Rightarrow \ln y = \ln 3 + 2\ln(x)$

$\Rightarrow \ln(y) = 2\ln(x) + \ln(3)$

$Y = mX + c$

$\therefore$ If we plot $\ln y$ against $\ln(x)$ then we should obtain a straight line with gradient 2 and y-intercept $\ln(3)$

b. $y = ab^x$

$$\ln y = \ln(ab^x) \Rightarrow \ln y = \ln(a) + \ln(b^x)$$

$$\Rightarrow \ln y = \ln(a) + x\ln(b)$$

$$Y = c + X\,m$$

$\therefore$ If we plot $\ln y$ against x then we should obtain a straight line with gradient $\ln(b)$ and y-intercept $\ln(a)$

[Direct students to work on Q1 in the worksheet]

Objective: Determining unknown constants using experimental data

Worked Example 2

A colony of bats is increasing. The population, P, is modelled by $P = a \times 10^{bt}$, where t is the time in years after 2000.

(i) Show that, according to this model, the graph of $\log_{10} P$ against t should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis. [3]

$$\log_{10} P = \log_{10}(a \times 10^{bt}) \Rightarrow \log_{10}(P) = \log_{10}(a) + \log_{10}(10^{bt})$$

$$\Rightarrow \log_{10}(P) = \log_{10}(a) + bt$$

$$Y = c + mX$$

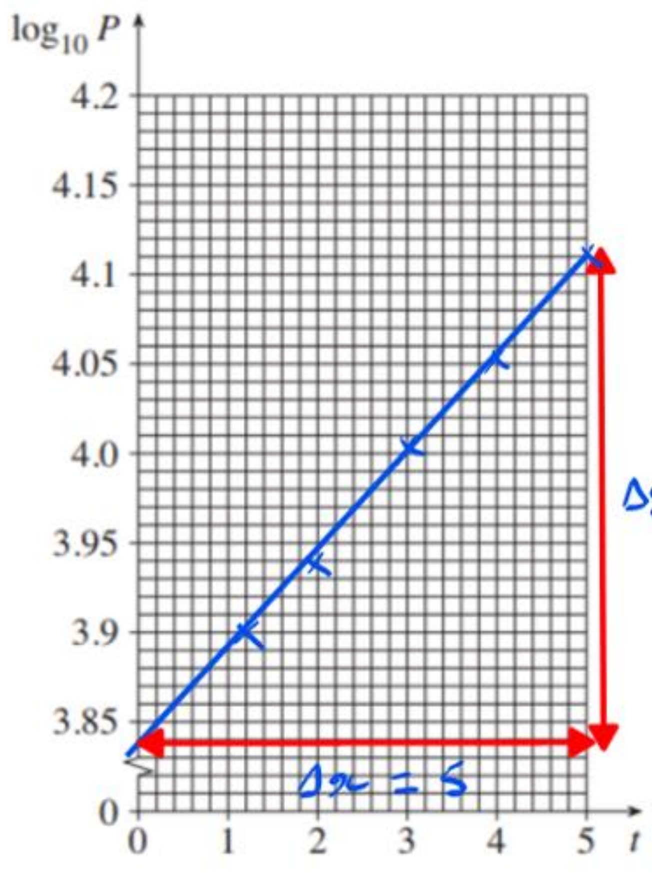
$\therefore$ Plotting $\log_{10}(P)$ against t will give a straight line of gradient b and y-intercept $\log_{10}(a)$

(ii) The table gives the data for the population from 2001 to 2005.

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800

Complete the table of values on the insert, and plot $\log_{10} P$ against t. Draw a line of best fit for the data. [3]

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800
$\log_{10} P$	3.90	3.94	4	4.05	4.11



$\Delta y = 4.11 - 3.84$

$= 0.27$

(iii) Use your graph to find the equation for P in terms of t. [4]

<u>Key Teaching Guidance</u>

- Some students will try to sub points from the table of data into the original exponential equation and try to find the value of the unknown constants that way. Stress that this approach is not correct as, due to the experimental nature of the data, it is not guaranteed that this point is not an outlier.
- Students should determine the y-intercept and gradient of the line of best fit from their hand-drawn graph, making the triangle for the gradient calculation as large as possible to minimise error.

$\log_{10}(P) = \log_{10}(a) + bt$

$Y = c + mX$

$\therefore m = b = \dfrac{0.27}{5} = 0.054$

$P = a \times 10^{bt}$,

$c = \log_{10}(a) = 3.84$

$\Rightarrow a = 10^{3.84} = 6918$

$\therefore P = 6918 \times 10^{0.054t}$

[Direct students to work on Q2-3 in the worksheet]

(Insert referred to in Worked Example 2)

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800
$\log_{10} P$