Conditional Probability Essential Practice

Skill:	Conditional Probability and Venn Diagrams

Questions

Attempt these questions independently showing full and clear solutions. Check each answer as you go.

1. A group of 80 people were tested for having one of three attributes: blue eyes, blonde hair and wearing glasses. The results are as follows:
   - 30 people wore glasses
   - 33 people had blue eyes
   - 40 people had blonde hair
   - 15 had blue eyes and blonde hair
   - 10 people had blue eyes and wore glasses
   - 17 people wore glasses and had blonde hair
   - 7 people had all three attributes

   a. Represent this information on a Venn diagram.
   b. A person was selected at random from the group. Determine the probability that this person:
      i. Had blue eyes, given that they wore glasses.
      ii. Had blue eyes, given that they didn't have blonde hair.
      iii. Didn't wear glasses, given that they didn't have blue eyes.
      iv. Had none of the three attributes, given that they weren't blonde.
      v. Had all three attributes given that they wore glasses and had blue eyes.
      vi. Had exactly one of the three attributes given that they were blonde.

2. Given that $P(A) = 0.3, P(B) = 0.45$ and $P(A \cap B) = 0.24$:
   a. Construct a Venn diagram representing these probabilities.
   b. Calculate the following probabilities:
      i. $P(A|B)$ ii. $P(A|B')$
      iii. $P(B'|A)$ iv. $P(A'|B')$
   c. Determine whether the events A and B are independent.

3. It is given that, for two events A and B:
   $$P(A|B) = 0.4 \quad P(B) = 0.25 \quad P(A) = \frac{1}{3}$$
   Calculate the following probabilities (y*ou may find a Venn diagram useful).*
   i. $P(A \cap B)$ ii. $P(B|A)$
   iii. $P(B|A')$ iv. $P(A \cup B|A')$

4. X and Y are two events such that $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{3}$ and $P(X \cup Y) = 0.9$.
   a. Use this information to calculate the following probabilities:
      i. $P(X \cap Y)$ ii. $P(Y)$
      iii. $P(X'|Y)$ iv. $P(X|Y')$
   b. Determine whether the events X and Y are independent.
   c. State how you know the events X and Y are not mutually exclusive.