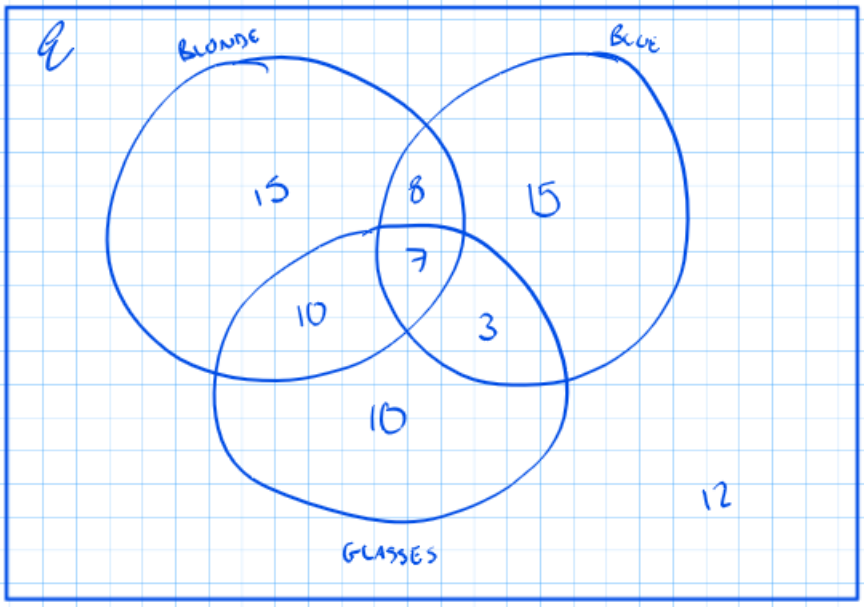
Conditional Probability Essential Practice Solutions

Skill: Conditional Probability and Venn Diagrams

Solutions

1 a/



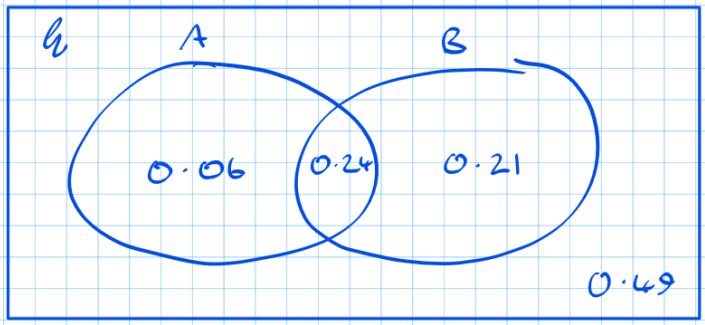
b/ i/ $\frac{7+3}{30} = \frac{1}{3}$ ii/ $\frac{15+3}{15+3+10+12} = \frac{9}{20}$

iii/ $\frac{15+12}{15+10+10+12} = \frac{27}{47}$

iv/ $\frac{12}{15+3+10+12} = \frac{3}{10}$ v/ $\frac{7}{7+3} = \frac{7}{10}$

vi/ $\frac{15}{15+10+8+7} = \frac{3}{8}$

2 a/



b/ i/ $P(A|B) = \frac{0.24}{0.24+0.21} = \frac{8}{15}$

ii/ $P(A|B') = \frac{0.06}{0.06+0.49} = \frac{6}{55}$

iii/ $P(B'|A) = \frac{0.06}{0.06+0.24} = \frac{1}{5}$

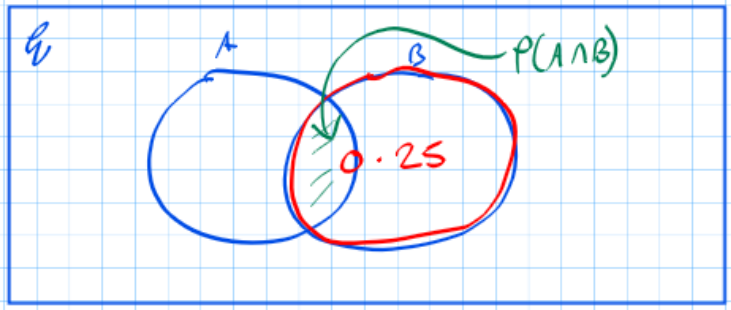
iv/ $P(A'|B') = \frac{0.49}{0.06+0.49} = \frac{49}{55}$

c/ $P(A \cap B) = 0.24$

$P(A) \times P(B) = 0.3 \times 0.45 = 0.135 \neq 0.24$

$\therefore$ Not independent

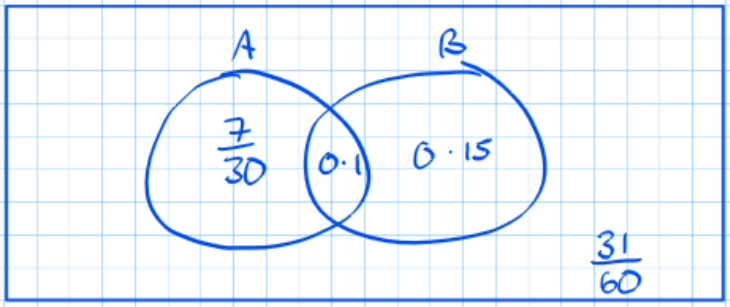
3



i/ $\frac{P(A \cap B)}{0.25} = 0.4$

$\Rightarrow P(A \cap B) = 0.4 \times 0.25$

$= 0.1$



ii/ $P(B|A) = \frac{0.1}{\frac{1}{3}}$

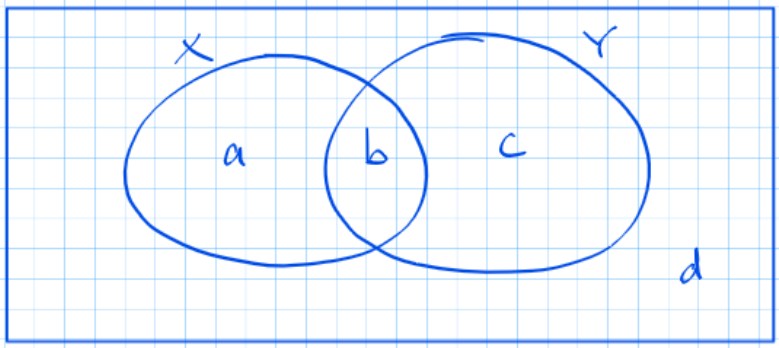
$= 0.3$

iii/ $P(B|A') = \frac{0.15}{0.15 + \frac{31}{60}} = \frac{9}{40}$

iv/ $\frac{0.15}{0.15 + \frac{31}{60}} = \frac{9}{40}$

[Bit of a trick Q as since we are given A' occurs, the A∪ is irrelevant]

4



$$P(X|Y) = \frac{b}{b+c} = \frac{1}{2} \Rightarrow 2b = b + c$$

$$\Rightarrow b = c \quad (A)$$

$$P(Y|X) = \frac{b}{a+b} = \frac{2}{3} \Rightarrow 3b = 2a + 2b$$

$$\Rightarrow b = 2a \quad (B)$$

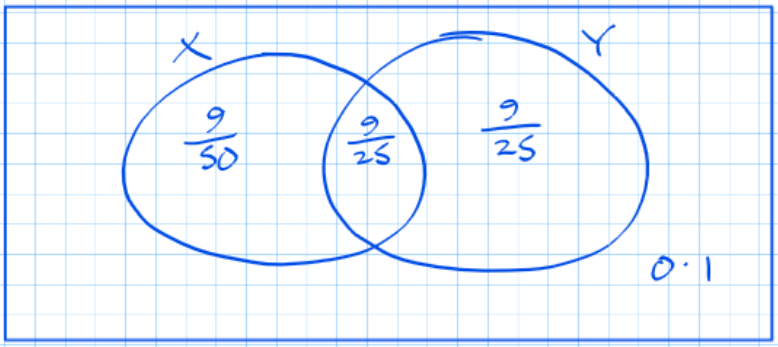
$$P(X \cup Y) = a + b + c = 0.9 \quad (C)$$

$$\Rightarrow (A), (B) \Rightarrow c = 2a$$

$$\therefore (C) \Rightarrow a + 2a + 2a = 0.9$$

$$\Rightarrow 5a = 0.9 \Rightarrow a = \frac{9}{50}$$

$$\Rightarrow b = \frac{9}{25} \Rightarrow c = \frac{9}{25} \Rightarrow d = 0.1$$



i/ $P(X \cap Y) = \frac{9}{25}$

ii/ $P(Y) = \frac{9}{25} + \frac{9}{25}$

$= \frac{18}{25}$

iii/ $P(X'|Y) = \frac{\frac{9}{25}}{\frac{9}{25} + \frac{9}{25}} = \frac{1}{2}$

iv/ $P(X|Y') = \dfrac{\frac{9}{50}}{\frac{9}{50} + 0.1} = \dfrac{9}{14}$

b/ $P(X \cap Y) = \frac{9}{25}$

$P(X) \times P(Y) = \left(\frac{9}{50} \times \frac{9}{26}\right) \times \frac{18}{25} = \frac{729}{15625} \neq \frac{9}{25}$

$\therefore$ Not independent

c/ $P(X \cap Y) \neq 0$ $\therefore$ not mutually exclusive.