Q1, (Jun 2014, Q2)
A is the point (1, 5) and B is the point (6, -1). M is the midpoint of AB. Determine whether the line with equation \( y = 2x - 5 \) passes through M.

\[ M = \left( \frac{1+6}{2}, \frac{5+(-1)}{2} \right) = \left( \frac{7}{2}, 2 \right) \]

Let \( x = \frac{7}{2} \) \( \Rightarrow \) \( y = 2 \left( \frac{7}{2} \right) - 5 = 7 - 5 = 2 \)

\[ \therefore \text{When } x = \frac{7}{2}, \ y = 2 \quad \therefore \text{line passes through } M \]

Q2, (Jun 2011, Q9)
A line \( L \) is parallel to the line \( x + 2y = 6 \) and passes through the point (10, 1). Find the area of the region bounded by the line \( L \) and the axes.

\[ x + 2y = 6 \quad \Rightarrow \quad 2y = 6 - x \quad \Rightarrow \quad y = 3 - \frac{1}{2}x \]

\[ \therefore \text{Grad} = -\frac{1}{2} \quad \text{Point} = (10, 1) \]

\[ \Rightarrow y - 1 = -\frac{1}{2}(x - 10) \]

\[ \Rightarrow y - 1 = -\frac{1}{2}x + 5 \quad \Rightarrow \quad y = -\frac{1}{2}x + 6 \]

\[ \text{y-inter: } (0, 6) \quad \text{x-inter: } y = 0 \Rightarrow 0 = -\frac{1}{2}x + 6 \Rightarrow \frac{1}{2}x = 6 \Rightarrow x = 12 \]

\[ A = \frac{1}{2} \times 6 \times 12 = 36 \]
Q3. (Jun 2016, Q10)

Prove that angle ABC is 90°.

\[ \text{Grad (AB)} = \frac{7 - 3}{2 - 0} = 2 \]
\[ \text{Grad (BC)} = \frac{-1 - 3}{8 - 0} = -\frac{4}{8} = -\frac{1}{2} \]
\[ 2 \times -\frac{1}{2} = -1 \]
\[ \therefore \text{AB and BC are perpendicular} \]
\[ \therefore \text{ABC = 90°} \]

Q4. (Jan 2006, Q7)

The line AB has equation \( y = 4x - 5 \) and passes through the point B(2, 3), as shown in Fig. 7. The line BC is perpendicular to AB and cuts the x-axis at C. Find the equation of the line BC and the x-coordinate of C.

\[ \text{Grad of AB} = 4 \Rightarrow \text{Grad of BC} = -\frac{1}{4}, \quad \text{Point} = (2, 3) \]
\[ \Rightarrow y - 3 = -\frac{1}{4}(x - 2) \Rightarrow 4y - 12 = -x + 2 \Rightarrow x + 4y - 14 = 0 \]
Let \( y = 0 \Rightarrow x - 14 = 0 \Rightarrow x \text{ coord of C} = 14 \]
Use coordinate geometry to answer this question. Answers obtained from accurate drawing will receive no marks.

A and B are points with coordinates (-1, 4) and (7, 8) respectively.

(i) Find the coordinates of the midpoint, M, of AB.

Show also that the equation of the perpendicular bisector of AB is $y + 2x = 12$. [6]

(ii) Find the area of the triangle bounded by the perpendicular bisector, the y-axis and the line AM, as sketched in Fig. 12.

\[
M = \left(\frac{-1+7}{2}, \frac{4+8}{2}\right) = (3, 6)
\]

\[
\text{Grad of \ AM} = \frac{8-4}{7-(-1)} = \frac{4}{8} = \frac{1}{2}
\]

\[
\text{Grad of \ perp \ bisector} = -2
\]

\[
y - 6 = -2(x - 3) \Rightarrow y - 6 = -2x + 6 \Rightarrow y + 2x = 12
\]

For \ line \ AB:

Point = (7, 8), \ Grad = \frac{1}{2}

\[
\Rightarrow y - 8 = \frac{1}{2}(x - 7)
\]

\[
\Rightarrow y - 8 = \frac{1}{2}x - \frac{7}{2}
\]

\[
\Rightarrow y = \frac{1}{2}x + \frac{9}{2}
\]

\[
\text{Area} = \frac{1}{2} (7-5)(3) = \frac{12}{4}
\]
(i) Points A and B have coordinates (−2, 1) and (3, 4) respectively. Find the equation of the perpendicular bisector of AB and show that it may be written as $5x + 3y = 10$.

\[
\text{Midpoint } = \left( \frac{-2+3}{2}, \frac{1+4}{2} \right) = \left( \frac{1}{2}, \frac{5}{2} \right)
\]

\[
\Rightarrow y - \frac{5}{2} = -\frac{5}{3} \left( x - \frac{1}{2} \right)
\]

\[
\Rightarrow 3y - \frac{15}{2} = -5x + \frac{5}{2}
\]

\[
\Rightarrow 5x + 3y = \frac{15}{2} + \frac{5}{2}
\]

\[
\Rightarrow 5x + 3y = 10
\]

(ii) Points C and D have coordinates (−5, 4) and (3, 6) respectively. The line through C and D has equation $4y = x + 21$. The point E is the intersection of CD and the perpendicular bisector of AB. Find the coordinates of point E.

Bisector of $AB: 5x + 3y = 10$

Line $CD: 4y = x + 21 \Rightarrow x = 4y - 21$

\[
\Rightarrow 5(4y - 21) + 3y = 10
\]

\[
\Rightarrow 20y - 105 + 3y = 10
\]

\[
\Rightarrow 23y = 115 \Rightarrow y = 5 \Rightarrow x = 4(5) - 21 = -1
\]

\[
\therefore E(-1, 5)
\]
Fig. 10 is a sketch of quadrilateral ABCD with vertices A (1, 5), B (-1, 1), C (3, -1) and D (11, 5).

(i) Show that AB = BC. [3]

(ii) Show that the diagonals AC and BD are perpendicular. [3]

(iii) Find the midpoint of AC. Show that BD bisects AC but AC does not bisect BD. [5]

\[
\begin{align*}
\text{i/ } AB &= \sqrt{(-1-1)^2 + (1-5)^2} = 2\sqrt{5} \\
\text{BC} &= \sqrt{(3-(-1))^2 + (-1-1)^2} = 2\sqrt{5}
\end{align*}
\]

\[
\therefore AB = BC
\]

\[
\text{iii/ }\text{Grad of } AC = \frac{-1-5}{3-1} = \frac{-6}{2} = -3
\]

\[
\text{Grad of } BD = \frac{5-1}{11-(-1)} = \frac{4}{12} = \frac{1}{3}
\]

\[
\frac{1}{3} x - 3 = -1 \therefore AC \text{ and } BD \text{ are perpendicular}
\]

\[
\text{iii/ }\text{MP of } AC = \left(\frac{1+3}{2}, \frac{5-1}{2}\right) = (2, 2)
\]

\[
\text{Grad of } BD = \frac{1}{3}
\]

\[
\Rightarrow y-1 = \frac{1}{3}(x-(-1))
\]

\[
\Rightarrow y-1 = \frac{1}{3}x + \frac{1}{3}
\]

\[
\Rightarrow y = \frac{1}{3}x + \frac{4}{3}
\]

Checking if (2, 2) is on BD: Let \(x = 2\)

\[
y = \frac{1}{3}(2) + \frac{4}{3} = \frac{6}{3} = 2
\]

\[
\therefore \text{BD bisects } AC
\]
Midpoint of $BD = \left(\frac{-1+1}{2}, \frac{1+5}{2}\right) = (0,3) \neq (2,2)$

\[\therefore AC \text{ does not bisect } BD\]