Q1, Jan 2006, Q2

(i) Simplify \((3x + 1)^2 - 2(2x - 3)^2\).

(ii) Find the coefficient of \(x^3\) in the expansion of

\[
(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1).
\]

\[
\begin{align*}
(3x + 1)(3x + 1) & - 2(2x - 3)(2x - 3) \\
& = (9x^2 + 3x + 3x + 1) - 2(4x^2 - 6x - 6x + 9) \\
& = 9x^2 + 6x + 1 - 2(4x^2 - 12x + 9) \\
& = 9x^2 + 6x + 1 - 8x^2 + 24x - 18 \\
& = x^2 + 30x - 17
\end{align*}
\]

\[
\begin{align*}
(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1) & \\
& = 4x^3 + 6x^3 + 2x^3 = 12x^3
\end{align*}
\]

\[
\therefore \text{Coefficient} = 12
\]

Q2, Jun 2006, Q4i

By expanding the brackets, show that

\[(x - 4)(x - 3)(x + 1) = x^3 - 6x^2 + 5x + 12.\]

\[
(x - 4)(x - 3)(x + 1) = (x^2 - 7x + 12)(x + 1) \\
= x^3 + x^2 - 7x^2 - 7x + 12x + 12 \\
= x^3 - 6x^2 + 5x + 12
\]

Q3, (Jun 2007, Q1)

Simplify \((2x + 5)^2 - (x - 3)^2\), giving your answer in the form \(ax^2 + bx + c\).

\[
(2x + 5)(2x + 5) - (x - 3)(x - 3) \\
= 4x^2 + 20x + 25 - (x^2 - 6x + 9) \\
= 4x^2 + 20x + 25 - x^2 + 6x - 9 = 3x^2 + 26x + 16
\]
Q4. (Jun 2007, Q5)

The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is $x$ metres.

(i) Show that the enclosed area, $A\text{ m}^2$, is given by

$$A = 20x - 2x^2.$$  \[2\]

\[
\text{Area} = x(20 - 2x) = 20x - 2x^2
\]

\[
* \text{(Since all three sides sum to 20)}
\]

\[
. \quad .
\]

Q5. (Jun 2008, Q6)

Expand and simplify $(x-5)(x+2)(x+5)$. \[3\]

\[
(x+5)(x-5)(x+2) = (x^2 - 25)(x+2) = x^3 + 2x^2 - 25x - 50
\]

Q6. (Jan 2012, Q3)

Given that

$$5x^2 + px - 8 = q(x - 1)^2 + r$$

for all values of $x$, find the values of the constants $p$, $q$ and $r$. \[4\]

\[
5x^2 + px - 8 = q(x^2 - 2x + 1) + r
\]

\[
\Rightarrow 5x^2 + px - 8 = qx^2 - 2qx + q + r
\]

Comparing coefficients,

\[
q = 5, \quad p = -2(5) = -10
\]

\[
q + r = -8 \Rightarrow r = -13
\]

\[
\therefore p = -10, \quad q = 5, \quad r = -13
\]
Q7, (Jan 2010, Q11, ii)
A lawn is to be made in the shape shown below. The units are metres.

(i) The perimeter of the lawn is \( P \) m. Find \( P \) in terms of \( x \). \([2]\)

(ii) Show that the area, \( A \) \( m^2 \), of the lawn is given by \( A = 9x^2 + 6x \). \([2]\)

\[ H_{YP} = \sqrt{(3x)^2 + (4x)^2} = \sqrt{25x^2} = 5x \]

\[ \therefore P = (2 + 5x) + 3x + (2 + x) = 3x \]

\[ \Rightarrow P = 14x + 4 \]

(i) Using \( A = \frac{1}{2}(a+b)h \) (area of a trapezium)

\[ A = \frac{1}{2} \left( 2x + 2 + 5x \right) (3x) \]

\[ = \frac{1}{2} \left( 6 + 6x \right) (3x) = \left( 2 + 3x \right) (3x) = 6x + 9x^2 \]

Q8, (Jun 2010, Q4ii)
Expand \((x - 2)^2(x + 1)\), simplifying your answer. \([3]\)

\[ (x^2 - 4x + 4)(x + 1) = x^3 + x^2 - 4x^2 + 4x + 4x + 1 \]

\[ = x^3 - 3x^2 + 1 \]

Q9, (Jan 2011, Q2)
Given that

\[ (x - p)(2x^2 + 9x + 10) = (x^2 - 4)(2x + q) \]

for all values of \( x \), find the constants \( p \) and \( q \). \([3]\)

\[ LHS = 2x^3 - 2px^2 + 9x^2 - 9px + 10x - 10p \]

\[ = 2x^3 + \left(-2p + 9\right)x^2 + (-9p + 10)x - 10p \]

\[ RHS = 2x^3 + qx^2 - 8x - 4q \]
Q10, (Jun 2012, Q1)
Simplify \((x - 5)(x^2 + 3) - (x + 4)(x - 1)\).

\[
x^3 + 3x - 5x^2 - 15 - (x^2 + 3x - 4)
= x^3 - 6x^2 - 11
\]

Q11, (Jan 2013, Q5)

(i) Simplify \((x + 4)(5x - 3) - 3(x - 2)^2\).

(ii) The coefficient of \(x^2\) in the expansion of
\[(x + 3)(x + k)(2x - 5)\]
is \(-3\). Find the value of the constant \(k\).

\[
\sqrt{5x^4 - 3x + 20x^6 - 12 - 3(x^2 - 4x + 4)}
\]
\[
= 5x^2 + 17x - 12 - 3x^2 + 12x - 12
= 2x^2 + 29x - 24
\]

\[
\text{Coeff of } x^2 \text{ is } -5 + 2k + 6 = 1 + 2k
\]
\[
1 + 2k = -3 \Rightarrow 2k = -4 \Rightarrow k = -2
\]
Q12. (Jun 2016, Q1)

(i) Simplify \((2x - 3)^2 - 2(3 - x)^2\).

\[
\begin{align*}
\text{i/ } & \quad (4x^2 - 12x + 9) - 2(9 - 6x + x^2) \\
& = 4x^2 - 12x + 9 - 18 + 12x - 2x^2 \\
& = 2x^2 - 9
\end{align*}
\]

(ii) Find the coefficient of \(x^3\) in the expansion of \((3x^2 - 3x + 4)(5 - 2x - x^3)\).

\[
\begin{align*}
\text{ii/ } & \quad (3x^2 - 3x + 4)(5 - 2x - x^3) \\
& \quad \quad - 6x^3 - 4x^3 = -10x^3
\end{align*}
\]

\[\therefore \text{ Coeff.} = -10\]