Using Central Limit Theorem (Form OCR 4733)

Q1, (Jun 2009, Q6)

The continuous random variable R has the distribution $N(\mu, \sigma^2)$. The results of 100 observations of R are summarised by

$$\Sigma r = 3360.0, \quad \Sigma r^2 = 115\,782.84.$$

(i) Calculate an unbiased estimate of μ and an unbiased estimate of σ^2. **[4]**

(ii) The mean of 9 observations of R is denoted by $\bar{R}$. Calculate an estimate of $P(\bar{R} > 32.0)$. **[4]**

(iii) Explain whether you need to use the Central Limit Theorem in your answer to part **(ii)**. **[2]**

Q2, (Jan 2012, Q4)

The discrete random variable H takes values 1, 2, 3 and 4. It is given that $E(H) = 2.5$ and $Var(H) = 1.25$. The mean of a random sample of 50 observations of H is denoted by $\bar{H}$.
Use a suitable approximation to find $P(\bar{H} < 2.6)$. **[5]**

Q3, (Jun 2012, Q2)

(i) For the continuous random variable V, it is known that $E(V) = 72.0$. The mean of a random sample of 40 observations of V is denoted by $\bar{V}$. Given that $P(\bar{V} < 71.2) = 0.35$, estimate the value of $Var(V)$. **[4]**

(ii) Explain why you need to use the Central Limit Theorem in part **(i)**, and why its use is justified. **[2]**

Q4, (Jun 2013, Q3)

The mean of a sample of 80 independent observations of a continuous random variable Y is denoted by $\bar{Y}$. It is given that $P(\bar{Y} \leqslant 157.18) = 0.1$ and $P(\bar{Y} \geqslant 164.76) = 0.7$.

(i) Calculate $E(Y)$ and the standard deviation of Y. **[6]**

(ii) State

(a) where in your calculations you have used the Central Limit Theorem,

(b) why it was necessary to use the Central Limit Theorem,

(c) why it was possible to use the Central Limit Theorem. **[3]**

Q5, (Jun 2018, Q4)

The discrete random variable Y has probability distribution given by

y	0	1	2	3
$P(Y = y)$	0.4	0.2	0.3	0.1

$\bar{Y}$ denotes the mean of 50 random independent observations of Y.

(i) Find the approximate distribution of $\bar{Y}$, giving the value(s) of any parameter(s). **[5]**

(ii) State the possible values taken by $\bar{Y}$ in the range from 1.4 to 1.5 inclusive. **[1]**
