Using Central Limit Theorem (Form OCR 4733)

**Q1, (Jun 2009, Q6)**

(i) \[
\frac{33.6}{\sqrt{100}} + 33.6^2 \times \frac{99}{100} = 29.16
\]

- B1 33.6 clearly stated [not recoverable later]
- M1 Correct formula used for biased estimate
- M1 \( \frac{100}{99} \), M’s independent. Eg \( \frac{\sum \beta}{99} \)
- A1 \( 33.6^2 \)
- SR B1 variance in range \([29.1, 29.2]\)

(ii) \( R \sim N(33.6, 29.16/9) \)

- M1 Normal, their \( \mu \), stated or implied
- A1 Variance [their (i)] \( \frac{99}{100} \)
- M1 Standardise & use \( \Phi \), 9 used, answer \( > 0.5 \), allow \( \sqrt{\text{errors}} \), allow cc 0.05 but \( \text{not} 0.5 \)
- A1 Answer, art 0.813

(iii) No, distribution of \( R \) is normal so that of \( \bar{R} \) is normal

- B2 2 Must be saying this. Eg "9 is not large enough": B0, Both: B1 max, unless saying that \( n \) is irrelevant.

**Q2, (Jan 2012, Q4)**

\[
\Phi \left( \frac{2.59 - 2.5}{0.025} \right) = \Phi(0.5692)
\]

- M1 Normal (any – can be implied by standardisation)
- A1 Mean 2.5
- A1 Variance or SD \( 1.25 \div 50 \) stated or used
- A1 Standardise 2.59 or 2.61, with \( \sqrt{(1.25/50)} \)
- A1 Answer in range \([0.715, 0.716]\) or \([0.736, 0.737]\) from 0.632

**Q3, (Jun 2012, Q3)**

(i) \[
\left( \frac{71.2 - 72.0}{\sigma / \sqrt{40}} \right) = -0.3853
\]

- M1 Standardise with \( \Phi^{-1} \) & \( \sqrt{40} \), allow cc, \( \sqrt{\text{errors}} \) eg \( \sigma^2 \)
- A1 Square roots and sign correct, no cc, no "1 - " error
- B1 \( z \) in range \( \pm 0.385, 0.386 \) seen
- B1 Final answer in range \([172, 173]\), or \( 13.1^2 \) cwo
- [4]

(ii) Parent distribution not known \( n \) is large

- B1 Or clear equivalent. Not "sample not normal" Or clear equiv. e.g. sample size > 30. Extras: max 1 "n large, \( n > n_0 \" \): B1 if \( n_0 \geq 30 \)
- [2]

RHS must be \( \Phi^{-1} \), i.e. \( \text{not} 0.7411 \) or 0.2589 or 0.6368 or 0.35. "1 - " error or \( \sqrt{40}/39 \): M1 A0 \( 0.674 \) may be from "1 - 0.35 = 0.75"

Needs variance, not SD

NB: Look out for \( -13.1 \rightarrow 172 \), M1 A0 B1 A0

Don’t bother about order of these statements. If numerical must be 30. Ignore “continuous”.
### Q4, (Jun 2013, Q4)

(i) \[
\frac{\mu - 157.18}{\sigma / \sqrt{80}} = 1.282; \quad \frac{\mu - 164.76}{\sigma / \sqrt{80}} = 0.5244
\]

Solve simultaneously: \[\mu = 170\]
\[\sigma = 89.44\]

- **M1:** Standardise once with \(\sqrt{80}\) or 80 and \(z\) signs may be wrong, allow “1−” errors
- **A1:** Both correct including signs, no cc
- **B1:** 1.28(155) seen anywhere, correct to 3 SF
- **B1:** [0.524, 0.525] seen anywhere
- **A1:** \(\mu\), a.r.t. 170 to 3 SF (169.98)
- **A1:** \(\sigma\), in range [89, 90], not isw
  - Don’t allow surds, e.g. 40\(\sqrt{5}\)
- **A0A0:** Allow cc, but not 0.1, 0.7, 0.9, 0.3 or \(\Phi(\text{these})\) [= .5398, .758, .8159, .6179]
- **B0:** \(z\) may be wrong (provided it is \(z\))
- **B0:** Ignore signs
- **B0:** Ignore signs
- **B0:** CWO\(\times2\) but allow from inaccurate \(z\) if answer(s) within limits. Look out for
  \(-89.44\) [must be in correct order, no repeats]

(ii) (a) In using normal tables
- **B1:** Any reference to \(\sigma/\sqrt{80}\): B0
- **B0:** No extras
- **B0:** Not “\(\geq 80\)”.

### Q5, (Jun 2018, Q5)

(i) \[E(Y) = \Sigma yP(Y = y) \text{ [\(= 1.1\)]} \]
\[\text{Var}(Y) = \Sigma y^2P(Y = y) - 1.1^2 = 2.3 - 1.1^2 = 1.09\]
Normal,
- **M1:** Allow if \(\Sigma P(Y = y)\) wrongly evaluated. Not for 1.1/50 if this is used to find var
- **A1:** Exact only, can be implied
- **M1:** Expect to see N(1.1, 0.0218)
- **A1:** FT on their \(E(Y)\), numerical value needed
- **B1:** FT on their \(\text{Var}(Y)\), numerical value needed as final answer, but allow “1.09/50”.
- **B1:** Not from binomial unless explicitly “variance”

(ii) 1.4, 1.42, 1.44, 1.46, 1.48, 1.5
- **B1:** These only, but allow omission of 1.4 and 1.5 [1]