The Exponential Distribution MS (From AQA MS04)

Q1, (Jun 2008, Q4)

(a)(i)	$\mathrm{F}(x) = 1 - \mathrm{e}^{-\frac{x}{200}}$			May be quoted
	$\mathrm{P}(X < 120) = 1 - \mathrm{e}^{-0.6}$	M1		
	$= 0.451$	A1	2	AWRT
(ii)	$\mathrm{P}(X > 160) = \mathrm{e}^{-0.8}$	M1		
	$= 0.449$	A1	2	AWRT
(iii)	$\mathrm{P}(X < 160 \mid X > 120)$			
	$= \dfrac{1 - [0.4512 + 0.4493]}{1 - 0.4512}$	M1 A1		or $= \mathrm{P}(X < 40)$ $= 1 - \mathrm{e}^{-0.2}$
	$= 0.181$	A1	3	AWRT
(b)	$1 - \mathrm{e}^{-\frac{m}{200}} = 0.5$			
	$\Rightarrow \mathrm{e}^{-\frac{m}{200}} = 0.5$	M1		
	$\Rightarrow m = \ln 0.5 \times (-200)$	M1		
	$= 139$ hours	A1	3	AWRT
	Total		**10**	

Q2, (Jun 2009, Q7)

(a)	$F(x)=1-e^{-\lambda x}$, $x \geq 0$	B1		$F(x)=1-e^{-\lambda x}$ B1B0
	$F(x)=0$, $x<0$	B1	2	Dependent
(b)	$1-e^{-\lambda x}=\frac{3}{4}$	M1		For either Q_1 or Q_3
	$Q_3=\frac{1}{\lambda}\ln 4$	m1A1		m1 for attempting to solve for either Q_1 or Q_3
	$1-e^{-\lambda x}=\frac{1}{4}$			
	$Q_1=\frac{1}{\lambda}\ln\frac{4}{3}$	A1		
	$\text{IQR}=\frac{1}{\lambda}\ln 3$	A1	5	AG
(c)(i)	$E(X^2)=\int_0^\infty \lambda x^2 e^{-\lambda x}\,dx$	M1		Limits required
	$=\left[-x^2e^{-\lambda x}\right]_0^\infty+\int_0^\infty 2xe^{-\lambda x}\,dx$	A1		
	$=\left[-\frac{2x}{\lambda}e^{-\lambda x}\right]_0^\infty+\int_0^\infty \frac{2}{\lambda}e^{-\lambda x}\,dx$	A1		
	$=\left[-\frac{2}{\lambda^2}e^{-\lambda x}\right]_0^\infty$	A1	4	
	$=\frac{2}{\lambda^2}$			AG
(ii)	$\text{Var}(X)=\frac{2}{\lambda^2}-\frac{1}{\lambda^2}=\frac{1}{\lambda^2}$	B1	1	AG
(d)(i)	$\frac{1}{\lambda}\ln 3=\frac{4}{\lambda^2}$	M1		
	$\lambda=\frac{4}{\ln 3}$	A1	2	
(ii)	$\text{IQR}\to 0$ as $\lambda\to\infty$	E1	1	
	Total		**15**	
	TOTAL		**75**	

Q3, (Jun 2011, Q6)

(a)(i)	$\mu = \mathrm{E}(X) = \int_0^\infty kxe^{-kx}\mathrm{d}x$	M1		
	$= \left[-xe^{-kx}\right]_0^\infty + \int_0^\infty e^{-kx}\mathrm{d}x$	M1		
	$= 0 + \left[\dfrac{e^{-kx}}{-k}\right]_0^\infty$	A1		
	$= \dfrac{1}{k}$		3	AG
(ii)	$\int_0^m ke^{-kx}\mathrm{dx} = \dfrac{1}{2}$	M1		**Or** $\mathrm{F}(x) = 1 - e^{-kx}$
	$\left[-e^{-kx}\right]_0^m = \dfrac{1}{2}$			
	$-e^{-km} + 1 = \dfrac{1}{2}$	A1		$\dfrac{1}{2} = 1 - e^{-km}$
	$e^{-km} = \dfrac{1}{2} \Rightarrow km = \ln 2 \Rightarrow m = \dfrac{\ln 2}{k}$	M1A1		
	$\ln 2 < 1 \Rightarrow m < \mu$	A1	5	
(b)(i)	$\mathrm{P}(T = 0) = e^{-\frac{t}{\lambda}}$	B1	1	
(ii)(A)	$\mathrm{P}(T > t) = 1 - \mathrm{F}(t) = e^{-\frac{t}{\lambda}}$	M1		
	$\mathrm{P}(T < t) = \mathrm{F}(t) = 1 - e^{-\frac{t}{\lambda}} \quad t \geq 0$	A1	2	AG
(B)	$\mathrm{f}(t) = \mathrm{F}'(t) = \dfrac{1}{\lambda}e^{-\frac{t}{\lambda}} \quad t \geq 0$	M1		
	$\Rightarrow$ Exponential distribution	A1	2	
		Total	**13**	

Q4, (Jun 2013, Q4)

(a)	$1-e^{-\frac{1}{\mu}x}=\frac{1}{4}$, $\frac{3}{4}$	M1		Either.
	$Q_1=\mu\ln\left(\frac{4}{3}\right)$	M1A1		M1 for attempting either Q_1 or Q_3.
	$Q_3=\mu\ln 4$	A1		
	$IQR=\mu\ln 3$	A1	5	CAO
(b)	$E\left(X^2\right)=\int_0^{\infty}\frac{1}{\mu}x^2e^{-\frac{1}{\mu}x}dx$	M1		Knowledge of formula.
	$=\left[-x^2e^{-\frac{1}{\mu}x}\right]_0^{\infty}+\int_0^{\infty}2xe^{-\frac{1}{\mu}x}dx$	M1A1		Using integration by parts.
	$=0+2\mu.\mu=2\mu^2$ (AG)	A1	4	
(c)	$SD=\sqrt{2\mu^2-\mu^2}=\mu$	B1		
	$\ln 3>1\Rightarrow$ SD < IQR	M1A1	3	
	Total		**12**	

Q5, (Jun 2012, Q5)

(a)	$\mathrm{E}(X^2)=\int_0^\infty kx^2\mathrm{e}^{-kx}\mathrm{d}x$	M1		
	$=\left[-x^2\mathrm{e}^{-kx}\right]_0^\infty+\int_0^\infty 2x\mathrm{e}^{-kx}\mathrm{d}x$	M1		
	$=0+\left[-\frac{2x}{k}\mathrm{e}^{-kx}\right]_0^\infty+\int_0^\infty\frac{2}{k}\mathrm{e}^{-kx}\mathrm{d}x$	A1		0 may be omitted, or limits inserted at end of process. ($\mathrm{E}(X)$ integral can be quoted.)
	$=0+\left[-\frac{2}{k^2}\mathrm{e}^{-kx}\right]_0^\infty$	A1		Ditto.
	$=\frac{2}{k^2}$	A1		
	$\mathrm{Var}(X)=\frac{2}{k^2}-\left(\frac{1}{k}\right)^2=\frac{1}{k^2}$	A1√	6	Their $\mathrm{E}(X^2)$ minus mean^2, provided **positive**. **SC** Allow B1 for those who write correct working and result, having failed to integrate correctly.
(b)(i)	$\mathrm{F}(x)=\int_0^x k\mathrm{e}^{-ku}\mathrm{d}u$	M1		
	$=\left[-\mathrm{e}^{-ku}\right]_0^x \quad =1-\mathrm{e}^{-kx}$	A1A1	3	
(ii)	$\left[1-\mathrm{e}^{-kx}\right]_0^N=0.9\Rightarrow\mathrm{e}^{-kN}=0.1$	M1		
	$\Rightarrow N=\frac{1}{k}\ln 10$	M1A1	3	M1 for taking logs. cao, acf
(c)	Mean $=a=\frac{1}{k}\Rightarrow k=\frac{1}{a}$, Mean $=3a=\frac{1}{k}\Rightarrow k=\frac{1}{3a}$	M1A1		
	$\mathrm{e}^{-\frac{1}{a}a}\cdot\mathrm{e}^{-\frac{1}{3a}a}=\mathrm{e}^{-1}\cdot\mathrm{e}^{-\frac{1}{3}}=\mathrm{e}^{-\frac{4}{3}}$	M1A1	4	cwo
	Total		**16**	

Q6, (Jun 2014, Q1)

(a)	$F(t) = \int_0^t 5e^{-5t}dt = [-e^{-5t}]_0^t$ $= 1 - e^{-5t} \quad t \geq 0$	M1A1 A1		If result quoted without proof award B1. Incorrect notation A0, unless recovery is clear. Need not see $t \geq 0$ for A1 .
	F(t) = 0 otherwise, or t < 0.	B1	4	
(b)	$1 - (1 - e^{-1}) = e^{-1}$ (0.368)	B1	1	
(c)	$e^{-5c} = 0.05 \Rightarrow e^{5c} = 20$ $\Rightarrow c = \frac{1}{5}\ln 20$ (0.599)	M1 A1	2	Accept 0.6. Some attempt to simplify a logarithmic answer is required.
	Total		**7**	

Q7, (Jun 2015, Q4)

(a) (i)(A)	$E(X) = \int_0^\infty x\frac{1}{\theta}e^{-\frac{x}{\theta}}dx$	M1		Used; ignore limits
	$= \left[-xe^{-\frac{x}{\theta}}\right]_0^\infty - \int_0^\infty -e^{-\frac{x}{\theta}}dx$	m1		Integration by parts; ignore limits
	$= 0 + \theta \times \int f(x)dx = \theta \times 1 \quad = \underline{\theta}$	A1	**(3)**	Fully correct convincing derivation
(B)	$P(X > x) = \int_x^\infty \frac{1}{\theta}e^{-\frac{x}{\theta}}dx = \left[-e^{-\frac{x}{\theta}}\right]_x^\infty$ **or** $P(X < x) = \int_0^x \frac{1}{\theta}e^{-\frac{x}{\theta}}dx = \left[-e^{-\frac{x}{\theta}}\right]_0^x$	M1		Correct integration with limits
	$= \left(0 + e^{-\frac{x}{\theta}}\right)$ **or** $1 - \left(-e^{-\frac{x}{\theta}} + 1\right) \quad = \underline{e^{-x/\theta}}$	A1	**(2)**	Fully correct convincing derivation
Note	1 Use of $1 - F(x) = 1 - (1 - e^{-x/\theta}) = e^{-x/\theta} \Rightarrow$ M0 A0			
			5	
(ii)	$P(m < X \leq \mu) = 0.5 - e^{-1}$ $= 0.5 - 0.36788 = \underline{\mathbf{0.132}}$	M1 A1	**2**	 AWRT (0.13212)
(b) (i)	$P(250 < L < 1250) = P(0.25 < X < 1.25)$	M1		Use of correct values of X
	$= e^{-\frac{0.25}{2}} - e^{-\frac{1.25}{2}}$	A1		OE
	$= 0.88250 - 0.53526 = \underline{\mathbf{0.347}}$	A1	**3**	AWRT (0.34724)

(ii)	$P(F_0) = P(X > 250) = \mathbf{0.88250}$	BF1		F on (b)(i)
	$P(F_0 \text{ in} \geq 5 \text{ in } 6) = 6 \times 0.8825^5 \times 0.1175 + 0.8825^6$	M1 AF1		At least one binomial term F on (b)(i) plus both binomial terms
	$= 0.37737 + 0.47237 = \underline{\mathbf{0.849 \text{ to } 0.85}}$	A1	4	AWFW (0.84974)
Notes	**1** B(6, 0.34724) ⇒ 1 − 0.97847 = 0.0215 ⇒ BF0 M1 A1F A0 **2** B(6, 0.53526) ⇒ 1 − 0.85397 = 0.1460 ⇒ BF0 M1 A1F A0 **3** B(6, *p*) ⇒ BF0 M1 A1F A0 only if seen correct expressions			
		Total	**14**	

Q8, (Jun 2016, Q1)

(a)	$P(X < 10) = 1 - e^{-10/16}$ **or** $1 - e^{-0.625}$	M1		Use of Exp(λ = 1/16 or 0.0625)
	$= \underline{\mathbf{0.46 \text{ to } 0.47}}$	A1	**(2)**	AWFW (0.46474)
(b)	$P(10 < X < 20) =$ $e^{-0.625} - e^{-1.25}$ **or** $\left(1 - e^{-1.25}\right) - (a)$	B1		Can be implied
	$= 0.53526 - 0.28650 = \underline{\mathbf{0.25}}$ **or** $= 0.71350 - 0.46474 = \underline{\mathbf{0.25}}$	B1	**(2)**	AWRT (0.24876)
(c)	$P(X \neq 15)$ = **<u>1 or one or unity or 100%</u>**	B1	**(1)**	CAO
			5	
		Total	**5**	

Q9, (Jun 2017, Q5)

(a) (i)	$f(x) = \frac{d}{dx}F(x) = \underline{\lambda e^{-\lambda x}}$	B1	1	CAO
(ii)	$E(X^2) = \int_0^\infty x^2(\lambda e^{-\lambda x})dx =$	M1		Ignore limits
	$\left[x^2(-e^{-\lambda x})\right]_0^\infty - \int_0^\infty 2x(-e^{-\lambda x})\,dx =$	A1		**Correct** integration by parts Ignore limits
	$\frac{2}{\lambda}\int_0^\infty x(\lambda e^{-\lambda x})\,dx$ $= \frac{2}{\lambda}E(X) = \underline{\frac{2}{\lambda^2}}$	A1	3	Fully **complete & correct** derivation to this point (OE) plus (OE) **correct** answer
(iii)	$\text{Var}(X) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$	Adep1	1	Dep on previous A1 AG
(b) (i)	$P(E < 15 \mid Po(12)) = 0.7720$ **or** 0.8444	M1		
	$= \underline{0.772}$	A1	2	AWRT (0.7720)
(ii) (A)	$P(T = 20) =$ **0 or zero or nought or nothing or nil**	B1	(1)	
(B)	$\underline{T \sim \text{Exp}(\lambda = 1/20)}$	B1		OE (PI); accept $\text{Exp}(\lambda_h = 3)$
	$P(T < 15) = 1 - e^{-15/20}$ **or** $1 - e^{-3/4}$ **or** $1 - 0.47237$	M1		
	$= \underline{0.527 \text{ to } 0.528}$	A1	(3)	AWFW (0.52763)
(C)	$P(15 < T < 25) =$ $(1 - e^{-25/20}) - (1 - e^{-15/20})$ **or** $[(1 - e^{-25/20}) - (B)]$ **or** $(e^{-15/20} - e^{-25/20})$	M1		$e^{-25/20} = e^{-3\times(5/12)}$ $e^{-15/20} = e^{-3\times(3/12)}$
	$= 0.71350 - 0.52763 = \underline{0.185 \text{ to } 0.186}$ **or** $= 0.47237 - 0.28650 = \underline{0.185 \text{ to } 0.186}$	A1	(2)	AWFW (0.18586)
			6	
		Total	13	