Continuous Random Variables (Mean, Variance and Median) (From OCR 4733)

Q1, (Jan 2006, Q8)

(i) \( \int_0^1 x^n \, dx = \left[ \frac{x^{n+1}}{n+1} \right]_0^1 = \frac{1}{n+1} \)  
\( \frac{k}{n+1} = 1 \) so \( k = n + 1 \)  
M1 \( \int \) limits 0 and 1  
M1 Equate to 1 and solve for \( k \)  
A1 Answer \( n + 1, \text{ not } [n+1] \), c.w.o.

(ii) \( \int_0^1 x^{n+1} \, dx = \left[ \frac{x^{n+2}}{n+2} \right]_0^1 = \frac{1}{n+2} \)  
\( \mu = \frac{k}{n+2} = \frac{n+1}{n+2} \)  
A1 Correctly obtain given answer  
M1 \( \int \) limits 0 and 1, not just \( x \cdot x^n \)  
M1 Answer \( \frac{1}{n+2} \)

(iii) \( \int_0^1 x^5 \, dx = \left[ \frac{x^6}{6} \right]_0^1 = \frac{1}{6} \)  
\( \sigma^2 = \frac{4}{6} - \left( \frac{1}{3} \right)^2 = \frac{2}{75} \)  
A1 Answer \( \frac{2}{75} \) or a.r.t. 0.027  
M1 \( \int \) limits 0 and 1, allow with \( n \)  
M1 Subtract \( \left( \frac{4}{3} \right)^2 \)

(iv) \( N\left( \frac{4}{3}, \frac{2}{7500} \right) \)  
B1 Normal stated  
B1 Mean \( \frac{4}{3} \) or \( \frac{n+1}{n+2} \)  
B1\( \sqrt{\text{ }} \) Variance their (iii)/100, a.e.f., allow \( \sqrt{\text{ }} \)  
B1\( \sqrt{\text{ }} \) Can be negative translation; or integration, must include correct method for integral  
A1\( \sqrt{\text{ }} \) (Their mean) \( -\frac{4}{3} \), c.w.d.  
B1\( \sqrt{\text{ }} \) Variance same as their (iii), or \( \frac{2}{75} \) by integration

Q2, (Jun 2007, Q7)

(i) \( S \) is equally likely to take any value in range. \( T \) is more likely at extremities  
B1 Positive parabola, symmetric about 0  
B1\( \sqrt{\text{ }} \) Completely correct, including correct relationship between two  
B1\( \sqrt{\text{ }} \) Don’t need vertical lines or horizontal lines outside range, but don’t give last B1 if horizontal line continues past “±1”

(ii) \( \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2 \, dx = \left[ \frac{x^3}{3} \right]_{-\frac{1}{2}}^{\frac{1}{2}} \)  
\( \frac{1}{2} \left( 1 - \frac{t^3}{2} \right) = 0.2 \) or \( \frac{1}{2} \left( t^3 + 1 \right) = 0.8 \)  
\( t^3 = 0.6 \)  
\( t = 0.8434 \)  
M1 \( \int \) recoverable if \( t \) used later  
B1 Correct indefinite integral  
M1 Equate to 0.2, or 0.8 if \([-1, l] \) used  
M1 Solve cubic equation to find \( t \)  
A1\( \sqrt{\text{ }} \) Answer, in range \([0.843, 0.844]\)
Q3, (Jun 2009, Q7i,ii)

(i) \[
\frac{2}{5} \int_0^3 x^2 (3-x) \, dx = \left[ \frac{3x^4}{4} - \frac{x^5}{5} \right]_0^3 = 2.7 - \left( \frac{9}{20} \right)^2 = \frac{9}{20} \text{ or } 0.45
\]

M1 Integrate \(x^2f(x)\) from 0 to 3 \([\text{not for } \mu]\)
A1 Correct indefinite integral
B1 Mean is 1\(\frac{1}{2}\), so \([\text{not recoverable later}]\)
M1 Subtract their \(\mu^2\)
A1 5 Answer 0.45

(ii) \[
\frac{2}{5} \int_0^{0.5} x(3-x) \, dx = \left[ \frac{3x^2}{2} - \frac{x^3}{3} \right]_0^{0.5} = \frac{2}{27} \text{ AG}
\]

M1 Integrate \(f(x)\) between 0, 0.5, must be seen somewhere
A1 2 Correctly obtain given answer \(\frac{2}{27}\), decimals other than 0.5 not allowed, 1 more line needed (eg \([\ ] = \frac{1}{2}\))

Q4, (Jun 2010, Q8)

(i) \[
\int_1^\infty kx^{-a} \, dx = \left[ \frac{kx^{-a+1}}{-a+1} \right]_1^\infty
\]

Correctly obtain \(k = a - 1\) AG

M1 Integrate \(f(x)\), limits 1 and \(\infty\) (at some stage)
B1 Correct indefinite integral
A1 3 Correctly obtain given answer, don’t need to see treatment of \(\infty\) but mustn’t be wrong. Not \(k^{a-1}\)

(ii) \[
\int_1^\infty 3x^{-2} \, dx = \left[ \frac{3x^{-1}}{-1} \right]_1^\infty = 1\frac{1}{2}
\]

M1 Integrate \(xf(x)\), limits 1 and \(\infty\) (at some stage)

M1 \([x^2 \text{ is not MR}]\)

M1 Integrate \(x^2f(x)\), correct limits
A1 Either \(\mu = 1\frac{1}{2}\) or \(E(X^2) = 3\) stated or implied, allow \(k, k/2\)
M1 Subtract their numerical \(\mu^2\), allow letter if subs later
A1 5 Final answer \(\frac{1}{4}\) or 0.75 only, e.g. not from \(\mu = -1\frac{1}{2}\).

[Sr: Limits 0, 1; can get (i) B1, (ii) M1M1M1]

(iii) \[
\int_1^2 (a-1)x^{-a} \, dx = \left[ -x^{-a+1} \right]_1^2 = 0.9
\]

\[
1 - \frac{1}{2^{a-1}} = 0.9, \quad 2^{a-1} = 10
\]

\(a = 4.322\)

M1* Equate \(f(x)\)dx, one limit 2, to 0.9 or 0.1.

[Normal: 0 ex 4]

dep* M1 Solve equation of this form to get \(2^{a-1} = \text{ number}\)

M1 indept Use logs or equivalent to solve \(2^{a-1} = \text{ number}\)
A1 4 Answer, a.r.t. 4.32. T&I: (M1M1) B2 or B0
Q5, (Jan 2012, Q7)

(i) \[ \int_1^4 \frac{1}{2\sqrt{x}} \, dx = \left[ \frac{1}{3} x^{\frac{3}{2}} \right]_1^4 = \frac{7}{3} \text{ or } 2.333 \ldots \]

M1 Attempt to integrate \( xf(x) \), correct limits
B1 Correct indefinite integral, a.e.f.
A1 Final answer \( \frac{7}{3} \) or equiv or a.r.t. \( 2.33 \)

(ii) \[ \int_1^{\sqrt{m} - 1} \frac{1}{2\sqrt{x}} \, dx = 0.5 \]

\[ m = 2.25 \]

M1 This or complementary integral, limits needed [not “- \( \infty \)"], equated to 0.5, needn’t attempt to evaluate
A1 This equation, any equivalent simplified form
A1 Answer 9/4 or exact equivalent only

Q6, (Jun 2013, Q5)

(i) \[ 1.5 \int_1^{\infty} y^{-2.5} y^2 \, dx = 1.5 \left[ \frac{y^{0.5}}{0.5} \right]_1^{\infty} \]

Upper limit gives infinite answer

M1 Attempt to integrate \( y^2 f(y) \), limits 1 and \( \infty \), allow any letter
B1 Correct indefinite integral \( [=3\sqrt{y}] \), ignore \( \mu [=3] \)
A1 Give correct reason, c.w.o. apart from constant, allow "\( = \infty \)"

(ii) \[ \int_0^3 x(x-a)^2 \, dx \]

\[ = \int_0^3 (x^3 - 2ax^2 + a^2 x) \, dx \]

\[ = \left[ \frac{3}{a^3} \left( \frac{x^4}{4} - 2 \frac{ax^3}{3} + a^2 x^2 \right) \right]_0^a \]

\[ = \frac{a}{4} \]

M1 Attempt this integral, correct limits seen somewhere
M1 Method for \[ x f(x) \], e.g. multiply out or parts, independent of first M1
A1 Correct form for integration, e.g. multiplied out correctly, or correct first stage of parts
B1 Correct indefinite integral
A1 \( \frac{a}{4} \) or exact equivalent (e.g. 0.25a) only

[scales/annotations not needed]

Touching axes (not asymptotic)
Don’t need vertical lines
i.e., 3/3 only if wholly right

Multiplication: needs 3 terms

E.g. \[ \frac{3}{a^3} x(x-a)^2 \]

\[ = \frac{3}{a^3} \left( \frac{x^4}{4} - 3 \frac{ax^3}{3} + \frac{3}{12} \right) \]
Q7. (Jun 2014, Q5)

(i) \[
\int_0^\frac{\pi}{2} \sin(\pi x) \, dx = \left[ -\frac{1}{2} \cos(\pi x) \right]_0^1 = \frac{1}{2} - \left( -\frac{1}{2} \right) = 1
\]
and function non-negative for all \(x\) in range

M1 Attempt to integrate \(f(x)\), limits (0, 1) somewhere, evidence e.g. “from calculator”
B1 Correctly integrate \(\sin(\pi x)\) to \(-\frac{1}{2} \cos(\pi x)\)
A1 Fully correct, need to see \(-\frac{1}{2} \cos(\pi x)\) and final 1, no wrong working seen
B1 Non-negative asserted explicitly, allow positive or equivalent. Not just graph drawn.

(Most will not get this mark!)

(ii) \[E(X) = \frac{1}{2}\]

M1 Correct shape, through 0, allow below axis outside range. Allow partial curve if clearly part of sine curve.
A1 Fully correct including no extension beyond [0, 1]. Don’t worry about grads at ends.
Ignore labelling of axes
B1 \(\frac{1}{2} or 0.5\), needs to be simplified, no working needed, no ft

(iii) \[
\int_0^1 \frac{1}{2} \pi \sin(\pi x) \, dx = 0.75; \left\{ -\frac{1}{2} \cos(\pi x) \right\}_0^1 = 0.75
\]
\(\cos(\pi q) = 0.5\)
Solve to get \(q = \frac{1}{3}\)

M1 Equate integral to correct probability, correct limits somewhere allow complementary probability (= 0.25) only if limits (0, q)
A1 \(\cos(\pi q) = 0.5\) or exact equivalent
A1 \(q = \frac{1}{3}\) or a.r.t. 0.333.

SR: Numerical (no working needed): 0.333 B3, 0.33 B2

(iv) \[
\int_0^1 \frac{1}{2} \pi x^2 \sin(\pi x) \, dx = \left( \frac{1}{2} \right)^2
\]

M1 Integral part correct, allow limits omitted, ignore \(dx\)
A1ft Subtract their \([E(X)]^2\), allow \(\mu\) in form of integral, correct limits needed, not just “\(\mu^2\)”

{note for scorers zoning – (ii) needs to be visible here}

(v) Values of \(x\) in range close to \(E(X)\) are more likely than those further away

B1 Need to see “values of \(x\)” or equivalent, and probably not “occur”
Not “the probability of \(x\) is greater when \(x\) is close to \(E(X)\)” etc. Not “PDF greater …”

[1]
### Q8, (Jun 2015, Q3)

<table>
<thead>
<tr>
<th>(i)</th>
<th>[ \int_{-3}^{3} \frac{3}{2a^3} x^2 , dx = \left[ \frac{x^3}{2a^3} \right]_{-3}^{3} = \frac{27}{a^3} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( = 0.125 ) so ( a = 6 )</td>
</tr>
</tbody>
</table>

| M1 dep* | Integrate, attempt at correct seen limits somewhere |
| B1     | Correct indefinite integral, can be implied by, e.g. \( \frac{27}{a^3} \) |
| *M1    | Equate, with limits, to 0.125 and solve |
| A1     | Solve to get \( a = 6 \) exactly |

**Allow e.g. “\( < 3 \)” = “\( \leq -4 \)”**

**Allow also for \( a^3 \) on top**

**Allow 6.00 but no other decimals. Not ±6**

| (ii) | \[ \mu = 0 \] |
|      | \[ \int_{-a}^{a} kx^4 \, dx = \left[ k \frac{x^5}{5} \right]_{-a}^{a} = \frac{3a^2}{5} \] |
|      | \( = 1.35 \) so \( a = 1.5 \) |

| B1   | Stated somewhere or calculated, any \( a \) |
| M1 dep* | Attempt to integrate \( x^2 f(x) \), limits ±\( a \) |
| B1   | Or exact equivalent, can be implied |
| *M1  | Equate to 1.35 and solve |
| A1   | \( a = 1.5 \pm 0.005 \), allow ±1.5, ignore “must be positive” |

**If \( \mu = 0 \) not mentioned anywhere, or “- \( \mu \)” stated [instead of “- \( \mu^2 \)”], B0 but can get remaining 4/5**

**Don’t need explicit \(-\mu^2\) here**

**NB: \( a = 3 \) is not MR but can get B1 for \( \mu = 0 \)**

<table>
<thead>
<tr>
<th>(iii)</th>
<th>( x ) is a value [values] that ( X ) takes</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Ignore irrelevancies or extra wrong, unless contradictory</td>
</tr>
</tbody>
</table>

**Not answers just about the function**
Q9, (Jun 2016, Q7)

(i) \[ x \text{ represents a} \text{ possible value(s) taken by } X \]

(iii) \[ \int_{1}^{\infty} ax^2 + bx^3 \, dx = \left[ -\frac{a}{2x^2} - \frac{b}{3x^3} \right]_1^{\infty} = \frac{a}{8} + \frac{b}{24} \]

Correct indefinite integral [from any set of limits or none]

M1 Integrate and substitute limits to obtain one expression

M1 Integrate and substitute limits to obtain a second expression

The limits must be two of \((1, \infty), (1, 2)\) or \((2, \infty)\), allow \((3, \infty)\) for \(\geq 2\)

M1 Equate two expressions from definite integrals to 1 or \(\frac{1}{16}\) or \(\frac{13}{16}\) as appropriate, and attempt to solve

A1 Both equations correct, any equivalent simplified form, can be implied

Correctly show \(a = 1\) AG, www

B1 7 Correct value of \(b\) obtained from at least one correct equation

SC: One equation only: M1B1 M0M0A0 A0B1, max 3/7

Two equations, assume \(a = 1\), solve for \(b\), checked in other equation: 7/7

M1 Integrate \(xf(x)\), limits 1 and \(\infty\) seen somewhere

B1fi Correct indefinite integral, their \(b\), can be implied by correct answer

Expect to see \[ \int_{1}^{\infty} x^2 + \frac{3}{2} x^3 \, dx = \left[ -\frac{1}{x} - \frac{3}{4x^2} \right]_1^{\infty} \]

A1 3 Correctly obtain 1\(\frac{3}{4}\) or a.r.t. 1.75 www, allow from calculator