### Q1, (Jan 2006, Q1i-iii)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>$F(t) = 1 - e^{-t/3}$ $(t &gt; 0)$</td>
<td>M1</td>
<td>attempt to solve, here or for 90th percentile. Depends on previous M mark.</td>
</tr>
<tr>
<td></td>
<td>For median $m$, $\frac{1}{2} = 1 - e^{-m/3}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\therefore e^{-m/3} = \frac{1}{2} \Rightarrow -\frac{m}{3} = \ln \frac{1}{2} = -0.6931$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow m = 2.079$</td>
<td>A1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>For 90th percentile $p$, $0.9 = 1 - e^{-p/3}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\therefore e^{-p/3} = 0.1 \Rightarrow -\frac{p}{3} = \ln 0.1 = -2.3026$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow p = 6.908$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>$f(t) = \frac{d}{dt} F(t) = \frac{1}{3} e^{-t/3}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>(for $t &gt; 0$, but condone absence of this)</td>
</tr>
<tr>
<td></td>
<td>$\mu = \int_{0}^{\infty} \frac{1}{3} t e^{-t/3} dt$</td>
<td>M1</td>
<td>Quoting standard result gets 0/3 for the mean.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1</td>
<td>attempt to integrate by parts</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{3} \left[ \left[ \frac{t e^{-t/3}}{-1/3} \right]<em>{0}^{\infty} + 3\int</em>{0}^{\infty} e^{-t/3} dt \right]$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= [0 - 0] + \left[ \frac{e^{-t/3}}{-1/3} \right]_{0}^{\infty} = 3$</td>
<td>A1</td>
<td>5</td>
</tr>
<tr>
<td>(iii)</td>
<td>$P(T &gt; \mu) = \left[ \text{from cdf} \right] e^{-\mu/3} = e^{-1}$</td>
<td>M1</td>
<td>[or via pdf]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>fit c’s mean ($&gt; 0$)</td>
</tr>
</tbody>
</table>
\[ f(x) = 12x^3 - 24x^2 + 12x, \quad 0 \leq x \leq 1 \]

(i) \[ E(X) = \int_0^1 xf(x) \, dx \]

\[ = 12 \left[ \frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} \right]_0^1 \]

\[ = 12 \left[ \frac{1}{5} - \frac{2}{4} + \frac{1}{3} \right] = 12 \times \frac{1}{30} = \frac{2}{5} \]

For mode, \( f'(x) = 0 \)

\[ f'(x) = 12(3x^2 - 4x + 1) = 12(3x - 1)(x - 1) \]

\[ \therefore f'(x) = 0 \text{ for } x = 1 \text{ and } x = \frac{1}{3} \]

Any convincing argument (e.g. \( f''(x) \)) that \( x = \frac{1}{3} \) (and not 1) is the mode.

(ii) \[ \text{CDF } F(x) = \int_0^x f(t) \, dt \]

\[ = 12 \left( \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} \right) \]

\[ = 3x^4 - 8x^3 + 6x^2 \]

\[ F(\frac{1}{3}) = \frac{3}{256} - \frac{8}{16} + \frac{9}{16} = \frac{3-32-24}{256} = \frac{67}{256} \]

\[ F(\frac{1}{2}) = \frac{3}{8} - \frac{8}{4} + \frac{4}{4} = \frac{3-16+24}{16} = \frac{11}{16} \]

\[ F(\frac{3}{4}) = \frac{3\times4}{256} - \frac{8}{64} + \frac{4}{16} = \frac{72-64+16}{256} = \frac{8}{256} \]

| M1 | Definition of cdf, including limits (or use of "+c" and attempt to evaluate it), possibly implied later. Some valid method must be seen. |
| A1 | Or equivalent expression; condone absence of domain [0, 1]. |
| B1 | For all three; answers given; must show convincing working (such as common denominator)! Use of decimals is not acceptable. |
\[ f(x) = k(1 - x) \quad 0 \leq x \leq 1 \]

(i) \[ \int_0^1 k(1 - x)\,dx = 1 \]
\[ \therefore k\left[ x - \frac{1}{2}x^2 \right]_0^1 = 1 \]
\[ \therefore k\left[ 1 - \frac{1}{2} \right] - 0 = 1 \]
\[ \therefore k = 2 \]

Labelled sketch: straight line segment from (0,2) to (1,0).

(ii) \[ E(X) = \int_0^1 2x(1-x)\,dx \]
\[ = \left[ x^2 - \frac{4}{3}x^3 \right]_0^1 = \left(1 - \frac{4}{3}\right) - 0 = \frac{1}{3} \]
\[ E(X^2) = \int_0^1 2x^2(1-x)\,dx \]
\[ = \left[ \frac{2}{3}x^3 - \frac{2}{4}x^4 \right]_0^1 = \left(\frac{2}{3} - \frac{1}{2}\right) - 0 = \frac{1}{6} \]
\[ \text{Var}(X) = \frac{1}{6} - \left(\frac{1}{3}\right)^2 \]
\[ = \frac{1}{18} \]

(iii) \[ F(x) = \int_0^x 2(1-t)\,dt \]
\[ = \left[ 2t - t^2 \right]_0^x = \left(2x - x^2 \right) - 0 = 2x - x^2 \]
\[ P(X > \mu) = P(X > \frac{2}{3}) = 1 - F\left(\frac{2}{3}\right) \]
\[ = 1 - \left(2 \times \frac{1}{3} - \left(\frac{1}{3}\right)^2 \right) = 1 - \frac{2}{3} = \frac{1}{3} \]

(iv) \[ F\left(1 - \frac{1}{\sqrt{2}}\right) = 2\left(1 - \frac{1}{\sqrt{2}}\right) - \left(1 - \frac{1}{\sqrt{2}}\right)^2 \]
\[ = 2 - \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}} - \frac{1}{2} = \frac{1}{2} \]

Alternatively:
\[ 2m - m^2 = \frac{1}{2} \]
\[ \therefore m^2 - 2m + \frac{1}{2} = 0 \]
\[ \therefore m = 1 \pm \frac{1}{\sqrt{2}} \]
\[ \text{SO } m = 1 - \frac{1}{\sqrt{2}} \]

<table>
<thead>
<tr>
<th>(i)</th>
<th>M1</th>
<th>Integral of ( f(x) ), including limits (possibly implied later), equated to 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E1</td>
<td>Convincingly shown. Beware printed answer.</td>
</tr>
<tr>
<td></td>
<td>G1</td>
<td>Correct shape.</td>
</tr>
<tr>
<td></td>
<td>G1</td>
<td>Intercepts labelled.</td>
</tr>
<tr>
<td>(ii)</td>
<td>M1</td>
<td>Integral for ( E(X) ) including limits (which may appear later).</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>M1</td>
<td>Integral for ( E(X^2) ) including limits (which may appear later).</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(iv)</td>
<td>M1</td>
<td>Substituting ( m = 1 - \frac{1}{\sqrt{2}} ) in ( c )'s cdf.</td>
</tr>
<tr>
<td></td>
<td>E1</td>
<td>Convincingly shown. Beware printed answer.</td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>Form a quadratic equation ( F(m) = \frac{1}{2} ) and attempt to solve it. ft</td>
</tr>
<tr>
<td></td>
<td>E1</td>
<td>( c )'s cdf provided it leads to a quadratic.</td>
</tr>
</tbody>
</table>
\[
f(x) = k(20 - x) \quad 0 \leq x \leq 20
\]

(a) \[ \int_0^{20} k(20 - x) \, dx = \left[ k \left( 20x - \frac{x^2}{2} \right) \right]_0^{20} = k \times 200 = 1 \]
\[
\therefore k = \frac{1}{200}
\]
Straight line graph with negative gradient, in the first quadrant.
Intercept correctly labelled (20, 0), with nothing extending beyond these points.
Sarah is more likely to have only a short time to wait for the bus.

(b) \[ \text{cdf} \quad F(x) = \int_0^x f(t) \, dt = \frac{1}{200} \left( 20t - \frac{t^2}{2} \right) = \frac{x}{10} - \frac{x^2}{400} \]
\[
P(X > 10) = 1 - F(10) = 1 - \left( 1 - \frac{1}{4} \right) = \frac{1}{4}
\]

(iii) Median time, \( m \), is given by \( F(m) = \frac{1}{2} \).
\[
\therefore \frac{m}{10} - \frac{m^2}{400} = \frac{1}{2}
\]
\[
\therefore m^2 - 40m + 200 = 0
\]
\[
\therefore m = 5.86
\]
**Q5, (Jun 2009, Q4i-iii)**

\[ f(x) = \frac{2x}{\lambda^2} \quad \text{for} \quad 0 < x < \lambda, \quad \lambda > 0 \]

(i) \( f(x) > 0 \) for all \( x \) in the domain.

\[
\int_0^\lambda \frac{2x}{\lambda^2} \, dx = \left[ \frac{x^2}{\lambda^2} \right]_0^\lambda = \frac{\lambda^2}{\lambda^2} = 1
\]

<table>
<thead>
<tr>
<th>M1</th>
<th>Correct integral with limits.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Shown equal to 1.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) 

\[
\mu = \int_0^\lambda \frac{2x^2}{\lambda^2} \, dx = \left[ \frac{2x^3}{3\lambda^2} \right]_0^\lambda = \frac{2\lambda^2}{3}
\]

\[
P(X < \mu) = \int_0^{\mu} \frac{2x}{\lambda^2} \, dx = \left[ \frac{x^2}{\lambda^2} \right]_0^\mu
\]

\[
= \frac{\mu^2}{\lambda^2} = \frac{4\lambda^2}{9} = \frac{4}{9}
\]

which is independent of \( \lambda \).

<table>
<thead>
<tr>
<th>M1</th>
<th>Correct integral with limits.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>c.a.o.</td>
</tr>
<tr>
<td>M1</td>
<td>Correct integral with limits.</td>
</tr>
<tr>
<td>A1</td>
<td>Answer plus comment. ft c's ( \mu ) provided the answer does not involve ( \lambda ).</td>
</tr>
</tbody>
</table>

(iii) Given \( \text{E}(X^2) = \frac{\lambda^2}{2} \)

\[
\sigma^2 = \frac{\lambda^2}{2} - \frac{4\lambda^2}{9} = \frac{\lambda^2}{18}
\]

<table>
<thead>
<tr>
<th>M1</th>
<th>Use of ( \text{Var}(X) = \text{E}(X^2) - \text{E}(X)^2 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>c.a.o.</td>
</tr>
</tbody>
</table>

2
Q6. (Jun 2010, Q4i,ii)

\[ f(x) = \lambda e^{-\lambda x} \text{ for } x \geq 0, \text{ where } \lambda > 0. \]

(i)

\[
\int_0^\infty f(x) \, dx = \int_0^\infty \lambda e^{-\lambda x} \, dx
\]

\[
= \left[- e^{-\lambda x}\right]_0^\infty
\]

\[
= (0 - (-e^0)) = 1
\]

M1 Integration of \( f(x) \).

M1 Use of limits or the given result.

A1 Convincingly obtained (Answer given.)

G1 Curve, with negative gradient, in the first quadrant only. Must intersect the y-axis.

G1 \((0, \lambda)\) labelled; asymptotic to x-axis.

(ii)

\[
E(X) = \int_0^\infty \lambda x e^{-\lambda x} \, dx
\]

\[
= \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}
\]

\[
E(X^2) = \int_0^\infty \lambda x^2 e^{-\lambda x} \, dx
\]

\[
= \frac{\lambda^2}{\lambda^3} = \frac{2}{\lambda^2}
\]

\[
\text{Var}(X) = E(X^2) - E(X)^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}
\]

M1 Correct integral.

A1 c.a.o. (using given result)

M1 Correct integral.

A1 c.a.o. (using given result)

M1 Use of \(E(X^2) - E(X)^2\)

A1
(ii) \[ E(X) = \frac{3}{16} \int_{0}^{b} (4x^2 - x^3) \, dx \]

\[ = \frac{3}{16} \left[ \left( \frac{4x^4}{4} - \frac{x^5}{5} \right) \right]_{0}^{b} \]

\[ = \frac{3}{16} \left( \frac{32}{3} - \frac{16}{4} - 0 \right) \]

\[ = \frac{4}{5} \]

\[ E(X^2) = \frac{3}{16} \int_{0}^{b} (4x^4 - x^5) \, dx \]

\[ = \frac{3}{16} \left[ x^5 - \frac{x^6}{6} \right]_{0}^{b} \]

\[ = \frac{3}{16} \left( 16 - \frac{32}{5} - 0 \right) \]

\[ = \frac{9}{5} \]

\[ \text{Var}(X) = \frac{9}{5} - \left( \frac{5}{4} \right)^2 = \frac{19}{80} \]

\[ \text{sd} = \sqrt{\frac{19}{80}} = 0.487 (\text{3}) \]
### Q8, (Jun 2011, Q3)

**Part (i)**

For the UQ $G(u) = 0.75$
\[
\therefore \left(1 + \frac{u}{200}\right)^{-2} = \frac{1}{4} \quad \therefore u = 200
\]
For the LQ $G(l) = 0.25$
\[
\therefore \left(1 + \frac{l}{200}\right)^{-2} = \frac{3}{4} \quad \therefore l = 200 \left(\frac{2}{\sqrt{3}} - 1\right) = 30.94...
\]
\[
\therefore \text{IQR} = 200 - 30.94 = 169.06
\]
For an outlier $x > \text{UQ} + 1.5 \times \text{IQR} = 200 + 1.5 \times 169 = 453.58 \approx 454 \text{ (nearest hour)}$

**Part (ii)**

For the c.d.f. $F(x)$
\[
\int_{0}^{x} \frac{1}{200} e^{\frac{t}{200}} dt
\]
\[
= \left[ -e^{\frac{x}{200}} \right]_{0}^{x} = \left( -e^{\frac{x}{200}} \right) - \left( -e^{0} \right) = 1 - e^{-\frac{x}{200}}
\]

- **M1** Use of $G(x)$ for either quartile.
- **A1** c.a.o.
- **A1** c.a.o.
- **M1** $\text{UQ} - \text{LQ}$
- **M1** $\text{UQ} + 1.5 \times \text{IQR}$
- **E1** Answer given; must be obtained genuinely.

- **M1** Correct integral, including limits (which may be implied subsequently).
- **A1** Correctly integrated.
- **E1** Limits used. Answer given; must be shown convincingly. Condone the omission of $x < 0$ part. Allow use of “+ c” with $F(0) = 0$.

- **M1** Use of $1 - F(x)$
- **E1** Answer given: must be convincing.

- **B1** Accept any form.
- **B1** Accept any form.
- **M1** Conditional probability. Not $P(X > 50) \times P(X > 400)$ unless clearly justified.
- **A1** Accept division of decimals, 3dp or better.

### Marks

<table>
<thead>
<tr>
<th>Part</th>
<th>Description</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Increasing curve, through $(0, 0)$, in first quadrant only.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Asymptotic behaviour.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Asymptote labelled; condone absence of axis labels.</td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td>Use of $G(x)$ for either quartile.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c.a.o.</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>$	ext{UQ} - \text{LQ}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$	ext{UQ} + 1.5 \times \text{IQR}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Answer given; must be obtained genuinely.</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>Correct integral, including limits (which may be implied subsequently).</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Correctly integrated.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Limits used. Answer given; must be shown convincingly. Condone the omission of $x &lt; 0$ part. Allow use of “+ c” with $F(0) = 0$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use of $1 - F(x)$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Answer given: must be convincing.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 0.7788(0)$</td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td>Accept any form.</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Accept any form.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conditional probability. Not $P(X &gt; 50) \times P(X &gt; 400)$ unless clearly justified.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Accept division of decimals, 3dp or better.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Accept a.w.r.t. 0.778 or 0.779.</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>Mean = 5/3 [ \therefore \lambda = 0.6 ]</td>
<td>B1 [1]</td>
</tr>
<tr>
<td>(iii)</td>
<td>$F(t) = \int_0^\infty 0.6e^{-0.6t} , dt$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \left[ -e^{-0.6t} \right]_0^\infty$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= (-e^{-\infty} - (-e^0)) = 1 - e^{-0.6t}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correct integral with limits (which may be implied subsequently). Allow use of “+ c” accompanied by a valid attempt to evaluate it.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly integrated.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Limits used or c evaluated correctly. Accept unsimplified form. If final answer is given in terms of $\lambda$ then allow max M1A1A0.</td>
<td>A1</td>
</tr>
<tr>
<td>(iv)</td>
<td>$P(T &gt; 1) = 1 - F(1)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= 1 - (1 - e^{-0.6}) = 0.5488$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ft c’s $F(t)$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cao Allow any exact form of the correct answer.</td>
<td>[2]</td>
</tr>
<tr>
<td>(v)</td>
<td>$F(m) = \frac{1}{2}$ [ \therefore 1 - e^{-0.6m} = \frac{1}{2} ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\therefore e^{-0.6m} = \frac{1}{2}$ [ \therefore -0.6m = -\ln 2 ] [ \therefore m = \frac{\ln 2}{0.6} ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use of definition of median. Allow use of c’s $F(t)$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Convincing attempt to rearrange to “$m = \ldots$”, to include use of logs.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Cao obtained only from the correct $F(t)$. Must be evaluated. Require 2 to 4 sf; condone 5.</td>
<td>A1</td>
</tr>
</tbody>
</table>
## Q10, (Jun 2014, Q4i-iii)

### (i) Shape

![Diagram](image)

**Solution:**

$$E \leq a$$ because the distribution is symmetrical about $$x = a$$

### (ii) Total area

$$\text{Total area} = \frac{1}{2} 2a \cdot ka$$

Or

$$\int_0^a kx \, dx + \int_a^{2a} k(2a - x) \, dx$$

$$k \left[ \frac{x^2}{2} \right]_0^a + k \left[ \frac{2ax - x^2}{2} \right]_a^{2a}$$

$$k \left( \frac{a^2}{2} - 0 \right) + k \left( 2a^2 - \frac{3a^2}{2} \right)$$

$$= ka^2$$

$$ka^2 = 1$$

$$k = \frac{1}{a^2}$$

**Attempt:**

- Correctly finding area in terms of $$k, a$$
- Equating area to 1 and convincingly obtaining result.

**Answer given:**

$$\text{Answer given}$$
Q11, (Jun 2016, Q3i-iv)

(iii) \[ \text{Var } X = k \int_0^a x^4 \, dx + k \int_0^a 2ax^2 - x^4 \, dx - a^2 \]

\[ k \left[ \frac{x^5}{5} \right]_0^a + k \left[ \frac{2ax^3}{3} - \frac{x^4}{4} \right]_0^a - a^2 \]

\[ \frac{a^2}{4} + \frac{16a^2}{3} - 4a^2 - \frac{2a^2}{3} + \frac{a^2}{4} - a^2 \]

\[ \text{Var } X = \frac{a^2}{6} \]

M1 Correct integral for \( E(X^2) \) including limits (which may appear later).

M1 Correctly integrated (dependent on M1 above).

M1 Using \( E(X^2) \).

A1 \( \text{cao} \) [4]

Q11, (Jun 2016, Q3i-iv)

i \[ k \int_{-1}^1 (1 - x^2) \, dx = 1 \to k \left[ x - \frac{x^3}{3} \right]_{-1}^1 = 1 \]

\[ \to \frac{4k}{3} = 1 \]

\[ \to k = \frac{3}{4} \]

M1 Correct integral including limits

M1 \( \text{(const)} \times k = 1 \)

A1 \( \text{cao} \) [3]

ii

[Diagram of f(x)]

B1 General shape between -1 and +1

B1 Axes labelled with scales and intercepts (FT their \( k \))

B1 Nothing outside \(|x| < 1\)

[3]

iii \[ E(X) = 0 \to V(X) = E(X^2) \]

\[ V(X) = \frac{3}{4} - \frac{1}{4} \int_{-1}^1 (x^2 - x^4) \, dx = \frac{3}{4} \left[ x^3 - \frac{x^5}{5} \right]_{-1}^1 \]

\[ = \frac{1}{5} \]

B1 for \( E(X) = 0 \)

M1 for correct integral including limits

A1 \( \text{cao} \) (ignore mistakes in working) [3]
\[ \frac{3}{4} \int_0^q (1 - x^2) \, dx = \frac{1}{4} \]

Integration

\[ \frac{3}{4} \left[ x - \frac{x^3}{3} \right] \]

\[ \rightarrow q - \frac{q^3}{3} = \frac{1}{3} \quad \text{or} \quad q^3 - 3q + 1 = 0 \]

\( g(0.345) = 0.006 \)
\( g(0.355) = -0.02 \)

Change of sign \( 0.345 < q < 0.355 \)

So upper quartile = 0.35 to 2 dp

M1 Correct limits and equality
B1 fit their \( k \)
A1 any correct simplified (3-term) cubic

M1 (allow correct alternative)
If solving using calculator: state all three solutions

E1 must be explained clearly
If solving by calculator: explain why only one works