Combinations of Continuous Random Variables

Q1, (OCR 4734, Jan 2009, Q1)

At a particular hospital, admissions of patients as a result of visits to the Accident and Emergency Department occur randomly at a uniform average rate of 0.75 per day. Independently, admissions that result from G.P. referrals occur randomly at a uniform average rate of 6.4 per *week*. The total number of admissions from these two causes over a randomly chosen period of four weeks is denoted by T. State the distribution of T and obtain its expectation and variance. **[4]**

Q2, (OCR 4734, Jan 2011, Q2)

In a Year 8 internal examination in a large school the Geography marks, G, and Mathematics marks, M, had means and standard deviations as follows.

	Mean	Standard deviation
G	36.42	6.87
M	42.65	10.25

Assuming that G and M have independent normal distributions, find the probability that a randomly chosen Geography candidate scores at least 10 marks more than a randomly chosen Mathematics candidate. Do not use a continuity correction. **[5]**

Q3, (OCR 4734, Jan 2013, Q1)

The independent random variables X and Y have the distributions $N(10, \sigma^2)$ and $Po(2)$ respectively. The random variable S is given by $S = 5X - 2Y + c$, where c is a constant.
It is given that $E(S) = Var(S) = 408$.

(i) Find the value of c and show that $\sigma^2 = 16$. **[4]**

(ii) Find $P(X \geqslant E(Y))$. **[2]**

Q4, (OCR 4734, Jun 2013, Q1)

The blood-test procedure at a clinic is that a person arrives, takes a numbered ticket and waits for that number to be called. The waiting times between the numbers called have independent normal distributions with mean 3.5 minutes and standard deviation 0.9 minutes. My ticket is number 39 and as I take my ticket number 1 is being called, so that I have to wait for 38 numbers to be called. Find the probability that I will have to wait between 120 minutes and 140 minutes. **[6]**

Q5, (OCR 4734, Jun 2014, Q1)

The independent random variables X and Y have Poisson distributions with parameters 16 and 2 respectively, and $Z = \frac{1}{2}X - Y$.

(i) Find $E(Z)$ and $Var(Z)$. **[3]**

(ii) State whether Z has a Poisson distribution, giving a reason for your answer. **[2]**

Q6, (OCR 4734, Jun 2015, Q1)

A laminate consists of 4 layers of material C and 3 layers of material D. The thickness of a layer of material C has a normal distribution with mean 1 mm and standard deviation 0.1 mm, and the thickness of a layer of material D has a normal distribution with mean 8 mm and standard deviation 0.2 mm. The layers are independent of one another.

(i) Find the mean and variance of the total thickness of the laminate. **[3]**

(ii) What total thickness is exceeded by 1% of the laminates? **[3]**

Q7, (OCR 4734, Jun 2015, Q5)

Two guesthouses, the Albion and the Blighty, have 8 and 6 rooms respectively. The demand for rooms at the Albion has a Poisson distribution with mean 6.5 and the demand for rooms at the Blighty has an independent Poisson distribution with mean 5.5. The owners have agreed that if their guesthouse is full, they will re-direct guests to the other.

(i) Find the probability that, on any particular night, the two guesthouses together do not have enough rooms to meet demand. **[3]**

(ii) The Albion charges £60 per room per night, and the Blighty £80. Find the probability, that on a particular night, the total income of the two guesthouses is exactly £400. **[4]**

(iii) If A is the number of rooms demanded at the Albion each night, and B the number of rooms demanded at the Blighty each night, find the mean and variance of the variable $C = 60A + 80B$. State whether C has a Poisson distribution, giving a reason for your answer. **[4]**

Q8, (OCR 4734, Jun 2016, Q1)

On a motorway, lorries pass an observation point independently and at random times. The mean number of lorries travelling north is 6 per minute and the mean number travelling south is 8 per minute. Find the probability that at least 16 lorries pass the observation point in a given 1-minute period. **[4]**

Q9, (OCR 4734, Jun 2015, Q5)

The independent random variables X and Y have distributions $N(30, \sigma^2)$ and $N(20, \sigma^2)$ respectively.
The random variable $aX + bY$, where a and b are constants, has the distribution $N(410, 130\sigma^2)$.

(i) Given that a and b are integers, find the value of a and the value of b. **[5]**

(ii) Given that $P(X > Y) = 0.966$, find σ^2. **[6]**

Q10, (OCR 4734, Jun 2017, Q4)

X, Y and Z are random variables. X and Y have independent Poisson distributions with means 2 and 3 respectively, and $Z = 4X + 5Y$.

(i) Find $E(Z)$ and $Var(Z)$. **[3]**

(ii) Explain how your answers to part **(i)** show that Z does not have a Poisson distribution. **[1]**

(iii) Find $P(Z = 15)$. **[4]**

Q11, (OCR 4734, Jun 2018, Q1)

The random variables X and Y have independent Poisson distributions with parameters 2 and 3 respectively, and $Z = 3X+4Y$. Find $P(Z = E(Z))$. **[5]**

Q12, (OCR 4734, Jun 2018, Q5)

A certain brand of ice cream is sold in cartons of different sizes. Large cartons contain ice cream whose mass is normally distributed with mean 412 g and standard deviation 10 g. Small cartons contain ice cream whose mass is normally distributed with mean 112 g and standard deviation 8 g.

(i) Find the probability that the total mass of ice cream in two randomly chosen large cartons and two randomly chosen small cartons is greater than 1 kg. **[4]**

(ii) Find the probability that the mass of ice cream in a randomly chosen large carton is greater than 4 times the mass of ice cream in a randomly chosen small carton. **[4]**

Q13, (OCR 4768, Jun 2014, Q1)

(i) Let X be a random variable with variance σ^2. The independent random variables X_1 and X_2 are both distributed as X. Write down the variances of $X_1 + X_2$ and $2X$; explain why they are different. **[3]**

A large company has produced an aptitude test which consists of three parts. The parts are called mathematical ability, spatial awareness and communication. The scores obtained by candidates in the three parts are continuous random variables X, Y and W which have been found to have independent Normal distributions with means and standard deviations as shown in the table.

	Mean	Standard deviation
Mathematical ability, X	30.1	5.1
Spatial awareness, Y	25.4	4.2
Communication, W	28.2	3.9

(ii) Find the probability that a randomly selected candidate obtains a score of less than 22 in the mathematical ability part of the test. **[3]**

(iii) Find the probability that a randomly selected candidate obtains a total score of at least 100 in the whole test. **[4]**

(iv) For a particular role in the company, the score $2X + Y$ is calculated. Find the score that is exceeded by only 2% of candidates. **[4]**

(v) For a different role, a candidate must achieve a score in communication which is at least 60% of the score obtained in mathematical ability. What proportion of candidates do not achieve this? **[3]**

Q14, (OCR 4768, Jun 2015, Q1)

A game consists of 20 rounds. Each round is denoted as either a starter, middle or final round. The times taken for each round are independently and Normally distributed with the following parameters (given in seconds).

Type of round	Mean	Standard deviation
Starter	200	15
Middle	220	25
Final	250	20

The game consists of 4 starter, 12 middle and 4 final rounds. Find the probability that

(i) the mean time per round for the 4 final rounds will exceed 260 seconds, **[3]**

(ii) all 20 rounds will be completed in a total time of 75 minutes or less, **[5]**

(iii) the 12 middle rounds will take at least 3.5 times as long in total as the 4 starter rounds, **[5]**

(iv) the mean time per round for the 12 middle rounds will be at least 25 seconds less than the mean time per round for the 4 final rounds. **[5]**
