Combinations of Continuous Random Variables

Q1, (OCR 4734, Jan 2009, Q1)

T has a Poisson distribution	B1	
$E(T)=28\times0.75+4\times6.4$	M1	From sum of Poissons
$= 46.6$	A1	
$Var(T)=46.6$	B1√ **4**	Ft $E(T)$ only if Poisson

Q2, (OCR 4734, Jan 2011, Q2)

Use $G - M \sim N(-6.23, \sigma^2)$	M1	**Or G-M-10~ N(-16.23, σ^2)**
$\sigma^2 = 6.87^2 + 10.25^2$	A1	
$z = (16.23)/\sigma$	M1	
$= 1.315$	A1	
Probability = 0.0942 or 0.0943	A1 **[5]**	Accept 0.094

Q3, (OCR 4734, Jan 2013, Q1)

(i)		$E(S)=50 - 4 + c = 408$	M1	Using $E(aX+bY+c)$	
		=> $c = 362$	A1		
		$Var(S)=25\sigma^2+8=408$	M1	Using $Var(aX+bY+c)$	
		=> $\sigma^2 = 16$ AG	A1 **[4]**		
(ii)		$P(X \geq 2)=P(Z\geq-8/4)$	M1		
		$=0.9772$	A1 **[2]**	$1 - \Phi(-2)$	

Q4, (OCR 4734, Jun 2013, Q1)

Total time $T \sim N(\mu, \sigma^2)$	M1	Using $\sum T_i \sim N$
$\mu = 38 \times 3.5$	A1	= 133 129.5 (from 37) or 136.5 (from 39) A0
$\sigma^2 = 38 \times 0.9^2$	A1	= 30.78 29.97 (from 37) or 31.59 (from 39) A0
$P(120 < T < 140) =$		M1 for standardising and combining . Allow even if spurious cc
$\Phi[(140 - 133)/\sigma] - \Phi[(120 - 133)/\sigma]$	M1	or σ^2 or from $38^2 \times 0.9^2$ used.
$= 0.8966 - 0.0095$	A1	allow 0.9724 – 0.0414 (from 37) or 0.7336 – 0.0017 (from 39) A1ft
$= 0.887$	A1 **[6]**	= 0.931 or = 0.732 A1ft

Q5, (OCR 4734, Jun 2014, Q1)

(i)		$E(Z) = 6$ $Var(Z) = ¼(16) + 2$ $= 6$	B1 M1 A1 **[3]**		
(ii)		No Difference between Poisson distributions is not Poisson, or Z may be fractional or negative.	B1 B1 **[2]**	Unless accompanied by a spurious reason. SC Allow B1 for 'no, you cannot subtract Poisson distributions'.	eg ft incorrect (i). Allow $Z \neq X+Y$

Q6, (OCR 4734, Jun 2015, Q1)

(i)		28 $4 \times 0.1^2 + 3 \times 0.2^2$ 0.16	B1 M1 A1 **[3]**	Not 4^2, 3^2	
(ii)		$\frac{x - "28"}{\sqrt{"0.16"}}$ $= 2.326$ 28.9	M1 B1 A1 **[3]**		

Q7, (OCR 4734, Jun 2015, Q5)

(i)		$A + B \sim Po(12)$ seen $1 - 0.7720$ 0.228	B1 M1 A1 **[3]**		

(ii)		A = 0 & B = 5 AND A = 4 & B = 2 identified $e^{-6.5} \times e^{-5.5}\frac{5.5^5}{5!} + e^{-6.5} \times \frac{6.5^4}{4!} \times e^{-5.5} \times \frac{5.5^2}{2!}$ 0.00717	B1 B1;B1 B1 **[4]**	These 2 pairs only. Allow B=5 alone (+A4,B2) for this mark. Each product seen (not nec added), or 0.0015x0.1714;0.11182x0.06181	Or 0.000258;0.00691. Allow from tables. Eg 0.5289-0.3575=0.1714
(iii)		Mean = 830 Var = $60^2 \times 6.5 + 80^2 \times 5.5$ = 58 600 Var ≠ Mean , so no.	B1 M1 A1 B1ft **[4]**	Any correct reason	eg Not all integer values possible.

Q8, (OCR 4734, Jun 2016, Q1)

Poisson identified or implied. Mean = 14 1 – 0.6694 0.331	B1 B1 M1 A1 **[4]**	If N used, B0.	

Q9, (OCR 4734, Jun 2016, Q5)

(i)		$30a + 20b = 410$ oe $a^2 + b^2 = 130$ oe $13a^2 - 246a + 1161 = 0$ $a = 9, b = 7$	B1 B1 M1A1 A1 **[5]**	Obtain quadratic in a or b.	$13b^2 - 164b + 511 = 0$
(ii)		$X - Y \sim N(10, 2\sigma^2)$ $\frac{0-10}{\sqrt{2\sigma^2}}$=-1.825 (or 6) (-)1.825/6 seen	M1A1 M1A1 B1	M1 for N(10,anything) must have 2σ².Must have matching signs for M1..	allow -10,30-20,20-30 for M1

Q10, (OCR 4734, Jun 2017, Q4)

(i)		23; $4^2\times2+5^2\times3$ $=107$	B1;M1 A1 **[3]**	32 or 75 oe seen M1	
(ii)		$E(Z)\neq Var(Z)$	B1 **[1]**	OR only $X+Y\sim Po$.	
(iii)		(0,3), identified $\frac{e^{-2}\times2^0}{0!}$ or $\frac{e^{-3}\times3^3}{3!}$ Both, multiplied 0.0303	B1 M1 M1 A1 **[4]**	allow '0' missing for this mark. fully correct method.	0.1353352x0.2240418

Q11, (OCR 4734, Jun 2018, Q1)

$E(Z)=18$ (2, 3) and (6, 0) identified. $\frac{e^{-2}\times2^2}{2!}\times\frac{e^{-3}\times3^3}{3!}$ or $\frac{e^{-2}\times2^6}{6!}\times\frac{e^{-3}\times3^0}{0!}$ both, added 0.0612	B1 B1 M1 M1 A1 **[5]**	0.0606 + 0.0006 may be from tables.

Q12, (OCR 4734, Jun 2018, Q5)

(i)		(N) (1048, 328) $\frac{1000-1048}{\sqrt{328}} = -2.650$ 0.996	B1B1 M1 A1 **[4]**	Units confusion M0
(ii)		(N) (–36 or 36, 1124) $\frac{36}{\sqrt{1124}} = 1.074$ 0.141(4)	B1B1 M1 A1 **[4]**	Units confusion M0. NOT mean and var from (i). Must have attempted to find new mean/var.

Q13, (OCR 4768, Jun 2014, Q1)

(i)		$\text{Var}(X_1+X_2)=2\sigma^2$ $\text{Var}(2X)=4\sigma^2$ X_1+X_2 means two independent values from X are added together. $2X$ means that one value from X is multiplied by 2.	B1 B1 E1 **[3]**	Allow 2Var(X) and 4Var(X) Any comment explaining why X_1+X_2 is different from $2X$	
(ii)		$\text{P}(X<22)=\text{P}\left(Z<\frac{22-30.1}{5.1}\right)$ $=\text{P}(Z<-1.5882)$ $=0.0561$	M1 A1 A1 **[3]**	For standardising. Award once, here or elsewhere Correct z value cao	
(iii)		$X+Y+W\sim\text{N}(83.7, 58.86)$ $\text{P}(X+Y+W>100)=\text{P}\left(Z>\frac{100-83.7}{\sqrt{58.86}}\right)$ $\text{P}(Z>2.1246)=0.0168$	B1 B1 M1 A1 **[4]**	Mean Variance (or sd=7.67) Correct set up cao	
(iv)		$2X\sim\text{N}(60.2, 104.04)$ $\rightarrow 2X+Y\sim\text{N}(85.6, 121.68)$ $\text{P}(2X+Y>b)=0.02$ $\rightarrow\frac{b-85.6}{\sqrt{121.68}}=2.054$ $\rightarrow b=108.26$ Score exceeded by 2% is 108.3	B1 B1 M1 A1 **[4]**	Variance 2.054 seen Correct set up cao	
(v)		$\text{P}(W<0.6X)=\text{P}(W-0.6X<0)$ $W-0.6X\sim\text{N}(10.14, 24.5736)$	M1 B1	Either way round Mean and variance	
		$\text{P}(W-0.6X<0)=\text{P}(Z<-2.0455)$ $=0.0204$	A1 **[3]**	Cao. Allow convincing recovery	

Q14, (OCR 4768, Jun 2016, Q1)

i		$F \sim N(250, 20^2)$ $P(\bar{F_4} > 260) = P\left(Z > \frac{260-250}{10}\right)$ $= P(Z > 1)$ $= 0.1587$	M1 M1 A1 [3]	standardisation including division by $\sqrt{n}$ correct tail (probability < 0.5) cao (to 3 or 4 sf)
ii		$(F_1 + F_2 + F_3 + F_4) = F' \sim N(1000, 1600)$ $(M_1 + M_2 + .. + M_{12}) = M' \sim N(2640, 7500)$ $(S_1 + S_2 + S_3 + S_4) = S' \sim N(800, 900)$ $\rightarrow T \sim N(4440, 100^2)$ $P(T < 4500) = P\left(Z < \frac{4500-4440}{100}\right) = P(Z < 0.6)$ $= 0.7258$	M1 A1 B1 M1 A1 [5]	for variances: at least one of 4×15^2 etc. seen allow '4×15 + 12×25 + 4×20', but not 4^2 etc. for 10,000 (or 2.778 in minutes) for 4440 (or 74 in minutes) correct tail (probability > 0.5) and √(their variance) art 0.726 (given to 3 or 4 sf)
iii		Looking for $P((M' - 3.5S') > 0)$ $[M' \sim N(2640, 7500)]$ $3.5S' \sim N(2800, 11025)$ $(M' - 3.5S') \sim N(-160, 18525)$ $= P\left(Z > \frac{160}{\sqrt{18525}}\right) = P(Z > 1.1755) = 0.1199$	M1 M1 B1,A1 A1 [5]	interpret the question correctly; e.g. '12M – 3.5×4S' or '12M > 3.5×4S' seen their Var(S')×12.25 mean and variance cao (0.1198 to 0.120)
iv		Looking for $P((\bar{F_4} - \bar{M_{12}}) > 25)$ $\bar{M_{12}} \sim N\left(220, \frac{625}{12}\right)$ $\bar{F_4} \sim N(250, 100)$ $(\bar{F_4} - \bar{M_{12}}) \sim N(30, 152.08)$ $P\left(Z > \frac{25-30}{\sqrt{152.08}}\right) = P(Z > -0.4054) = 0.6574$	M1 M1 A1 B1 A1 [5]	interpret the question correctly; e.g. $P(\bar{F_4} > \bar{M_{12}} + 25)$ seen variance: at least one of $\frac{25^2}{12}$ or $\frac{20^2}{4}$ seen correct variance correct mean answer rounds to 0.657 or 0.658