Matrices, Simultaneous Equations and Applications to Intersections of Planes (from OCR 4726)

Q1, (Jan 2007, Q10)

The matrix $\mathbf{D}$ is given by $\mathbf{D} = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$, where $a \neq 2$.

(i) Find $\mathbf{D}^{-1}$. [7]

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} ax + 2y \qquad &= 3, \\ 3x + y + 2z &= 4, \\ - y + z &= 1. \end{aligned}$$

[4]

(iii) Interpret your solution to (iii) geometrically, giving your answer in terms of the intersection of three planes. [1]

Q2, (Jun 2006, Q8)

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{M}$. [3]

(ii) Hence find the values of a for which $\mathbf{M}$ is singular. [3]

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + 4y + 2z &= 3a, \\ x + ay \qquad &= 1, \\ x + 2y + z &= 3, \end{aligned}$$

have any solutions when

(a) $a = 3$,

(b) $a = 2$.

[4]

(iv) Interpret your solution to (iii)(a) and (iii)(b) geometrically, giving your answer in terms of the intersection of three planes. [2]

Q3, (Jun 2009, Q9)

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of **A**. **[3]**

(ii) Hence find the values of a for which **A** is singular. **[3]**

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + y + z &= 2a, \\ x + ay + z &= -1, \\ x + y + 2z &= -1, \end{aligned}$$

have any solutions when

(a) $a = 0$,

(b) $a = 1$.

[4]

(iv) Interpret your solution to (iii)(a) and (iii)(b) geometrically, giving your answer in terms of the intersection of three planes. [2]

Q4, (Jan 2010, Q9)

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$, where $a \neq 1$.

(i) Find $\mathbf{A}^{-1}$. **[7]**

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} 2x - y + z &= 1, \\ 3y + z &= 2, \\ x + y + az &= 2. \end{aligned}$$

[4]

(iii) Interpret your solution to (ii) geometrically, giving your answer in terms of the intersection of three planes. [1]

Q5 (Jun 2012, Q10)

The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} a & 2 & -1 \\ 2 & a & 1 \\ 1 & 1 & a \end{pmatrix}$.

(i) Find the determinant of **D** in terms of a. **[3]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} ax + 2y - z &= 0 \\ 2x + ay + z &= a \\ x + y + az &= a \end{aligned}$$

For each of the following values of a, determine whether or not there is a unique solution. If the solution is not unique, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 2$

(c) $a = 0$ **[7]**

(iii) Interpret your solution to (ii)(a), (b) and (c) geometrically, giving your answer in terms of the intersection of three planes. [3]

Q6, (Jun 2015, Q9)

The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & a & 3 \\ 0 & 1 & a \end{pmatrix}$.

(i) Find the values of a for which **D** is singular. **[6]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} x + 3y + 4z &= 3 \\ 2x + ay + 3z &= 2 \\ y + az &= 0 \end{aligned}$$

For each of the following values of a, determine whether or not there is a unique solution. If a unique solution does not exist, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 1$

[4]

(iii) Interpret your solution to (ii)(a) and (b) geometrically, giving your answer in terms of the intersection of three planes. [2]