Matrices, Simultaneous Equations and Applications to Intersections of Planes (from OCR 4726)

Q1, (Jan 2007, Q10)

(i)	M1		Show correct expansion process for
	M1		3 x 3
$\Delta = \det \mathbf{D} = 3a - 6$	A1		Correct evaluation of any 2 x 2 det
	M1		Obtain correct answer
	A1		Show correct process for adjoint
	B1		entries
$\mathbf{D}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3 & -2 & 4 \\ -3 & a & -2a \\ -3 & a & a-6 \end{pmatrix}$	A1	7	Obtain at least 4 correct entries in adjoint
(ii) $\frac{1}{\Delta}\begin{pmatrix} 5 \\ 2a-9 \\ 5a-15 \end{pmatrix}$	M1		Divide by their determinant
	A1A1A1 ft all 3	4	Obtain completely correct answer
		11	
			Attempt product of form $\mathbf{D}^{-1}\mathbf{C}$, or eliminate to get 2 equations and solve
			Obtain correct answers, ft their inverse

(iii) The equations are consistent so the planes intersect at the point B1

$$\frac{1}{\Delta}\begin{pmatrix} 5 \\ 2a-9 \\ 5a-15 \end{pmatrix}$$

Q2, (Jun 2006, Q8)

(i)	M1		Correct expansion process shown
$a\begin{bmatrix} a & 0 \\ 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & a \\ 1 & 2 \end{bmatrix}$	A1		Obtain correct unsimplified expression
$a^2 - 2a$	A1	3	Obtain correct answer
(ii)	M1		Solve their det **M** = 0
$a = 0$ or $a = 2$	A1A1ft	3	Obtain correct answers
(iii) (a) det M ≠ 0 ∴ Solutions	B1 B1		Solution, as inverse matrix exists or **M** non-singular or det**M** $\neq 0$
(b) Infinite solutions since equations consistent	B1 B1	4	Solutions, eqn. 1 is multiple of eqn 3
		10	

(iv)(a) A single solution so planes meet at a point (B1)

(b) All terms of equation 1 are multiples of equations 3 so planes 1 and 3 are identical, therefore the planes intersect in a line.

Q3, (Jun 2009, Q9)

(i) $a\begin{vmatrix} a & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix}$	M1 A1		Correct expansion process shown Obtain correct unsimplified expression
$2a^2 - 2a$	A1	3	Obtain correct answer
(ii)	M1		
$a = 0$ or 1	A1ft A1ft	3	Equate their det to 0 Obtain correct answers, ft solving a quadratic
(iii) (a)	B1 B1		Equations consistent, but non unique solutions
(b)	B1 B1	4 **10**	Correct equations seen & inconsistent, no solutions

(iv)(a) Equations consistent so planes meet in a line (i.e. sheaf)

(b) Equations inconsistent so the three planes never intersect because planes 1 and 2 are parallel since the coefficients of x, y and z are multiples of each other (i.e. 1).

Q4, (Jan 2010, Q9)

(i)

	M1		Show correct expansion process for 3×3 or multiply adjoint by **A**
	M1		Correct evaluation of any 2×2 at any stage
$\det \mathbf{A} = \Delta = 6a - 6$	A1		Obtain correct answer
$\mathbf{A}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3a-1 & a+1 & -4 \\ 1 & 2a-1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$	M1		Show correct process for adjoint entries
	A1		Obtain at least 4 correct entries in adjoint
	B1		Divide by their determinant
	A1	**7**	Obtain completely correct answer

(ii) $\frac{1}{\Delta}\begin{pmatrix} 5a-7 \\ 4a-5 \\ 3 \end{pmatrix}$

	M1		Attempt product of form $\mathbf{A}^{-1}\mathbf{C}$ or eliminate to get 2 equations and solve
	A1A1A1 ft all 3		Obtain correct answer
		4	**S.C.** if det now omitted, allow max A2 ft
		11	

(iii) A single solution exists therefore the planes intersect at

$$\frac{1}{\Delta}\begin{pmatrix} 5a-7 \\ 4a-5 \\ 3 \end{pmatrix}$$

(Jun 2012, Q10)

(i)		$a^3 - 4a$	M1 M1 A1 **[3]**	Show correct expansion process for 3×3 Correct evaluation of any 2×2 Obtain correct answer
(ii)	(a)		B1 **[1]**	det **D = 15** so unique sol'n or solve to find correct solution (-2/5, 1, 4/5)
(ii)	(b)		B1 M1 A1 **[3]**	Their det **D** = 0, so non-unique solutions Attempt to solve equations with $a = 2$ Explain inconsistency with correct working
(ii)	(c)		B1 M1 A1 **[3]**	Their det **D** = 0, so non-unique solutions Attempt to solve equations with $a = 0$ Explain consistency with correct working

(iii)(a) Unique solutions so planes meet at a point.

(b) Equations inconsistent and planes non-parallel so the three planes never meet so form a triangular prism.

(c) Equations consistent so meet in a line and form a sheaf.

Q6, (Jun 2015, Q9)

(i)		$a^2 - 6a + 5$ $a = 5$ or 1	M1 M1 M1 A1 M1 A1 **[6]**	Attempt to find det **D** Show correct process for a 3×3, condone sign errors Show correct processes for a 2×2 Obtain correct answer Attempt to solve det **D** = 0 Obtain correct answers
(ii)	**(a)(b)**		B1 B1 M1 A1 **[4]**	State unique solution State non unique solutions Attempt to solve equations with $a = 1$ Explain inconsistency with correct working **S.C. Answer to (i) wrong, allow correct unique/non-unique B1ft, B1ft only**

(iii)(a) Unique solution so planes meet at a point

(b) Equations inconsistent and planes not parallel, therefore the three planes form a triangular prism.