Maclaurin Expansion Exam Question (From OCR 4726)

Q1, (Jan 2006, Q1)

(i) Write down and simplify the first three non-zero terms of the Maclaurin series for $\ln(1 + 3x)$. [3]

(ii) Hence find the first three non-zero terms of the Maclaurin series for

$$e^x \ln(1 + 3x),$$

simplifying the coefficients. [3]

Q2, (Jan 2007, Q1)

It is given that $f(x) = \ln(3 + x)$.

(i) Find the exact values of $f(0)$ and $f'(0)$, and show that $f''(0) = -\frac{1}{9}$. [3]

(ii) Hence write down the first three terms of the Maclaurin series for $f(x)$, given that $-3 < x \leqslant 3$. [2]

Q3, (Jan 2009, Q1)

(i) Write down and simplify the first three terms of the Maclaurin series for e^{2x}. **[2]**

(ii) Hence show that the Maclaurin series for

$$\ln(e^{2x} + e^{-2x})$$

begins $\ln a + bx^2$, where a and b are constants to be found. **[4]**

Q4, (Jan 2010, Q2)

It is given that $f(x) = \tan^{-1}(1 + x)$.

(i) Find $f(0)$ and $f'(0)$, and show that $f''(0) = -\frac{1}{2}$. **[4]**

(ii) Hence find the Maclaurin series for $f(x)$ up to and including the term in x^2. **[2]**

Q5, (Jan 2013, Q5)

You are given that $f(x) = e^{-x}\sin x$.

(i) Find $f(0)$ and $f'(0)$. **[3]**

(ii) Show that $f''(x) = -2f'(x) - 2f(x)$ and hence, or otherwise, find $f''(0)$. **[4]**

(iii) Find a similar expression for $f'''(x)$ and hence, or otherwise, find $f'''(0)$. **[2]**

(iv) Find the Maclaurin series for $f(x)$ up to and including the term in x^3. **[2]**

Q6, (Jun 2013, Q3)

It is given that $f(x) = \tanh^{-1}\left(\frac{1-x}{3+x}\right)$ for $x > -1$.

(i) Show that $f''(x) = \frac{1}{2(x+1)^2}$. **[6]**

(ii) Hence find the Maclaurin series for f(x) up to and including the term in x^2. **[4]**

Q7, (Jun 2014, Q2)

It is given that $f(x) = \ln(1+x^2)$.

(i) Using the standard Maclaurin expansion for $\ln(1+x)$, write down the first four terms in the expansion of f(x), stating the set of values of x for which the expansion is valid. **[3]**

(ii) Hence find the exact value of

$$1 - \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{3}\left(\frac{1}{2}\right)^4 - \frac{1}{4}\left(\frac{1}{2}\right)^6 + \dots.$$ **[2]**

Q8, (Jun 2015, Q2)

It is given that $f(x) = \ln(1+\sin x)$. Using standard series, find the Maclaurin series for f(x) up to and including the term in x^3. **[4]**

Q9, (Jun 2016, Q5)

It is given that $y = \tan^{-1}2x$.

(i) Find $\frac{dy}{dx}$ and show that $\frac{d^2y}{dx^2} + 4x\left(\frac{dy}{dx}\right)^2 = 0$. **[3]**

(ii) Find the Maclaurin series for y up to and including the term in x^3. Show all your working. **[4]**

(iii) The result in part **(ii)**, together with the value $x = \frac{1}{2}$, is used to find an estimate for π. Show that this estimate is only correct to 1 significant figure. **[2]**
