Lines and Planes (Equations, Angles Between and Intersection) (From OCR 4727)

Q1, (Jun 2007, Q2)
A line \( l \) has equation \( \mathbf{r} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) \) and a plane \( \Pi \) has equation \( 8x - 7y + 10z = 7 \). Determine whether \( l \) lies in \( \Pi \), is parallel to \( \Pi \) without intersecting it, or intersects \( \Pi \) at one point.

Q2, (Jun 2007, Q6)
Lines \( l_1 \) and \( l_2 \) have equations
\[
\frac{x - 3}{2} = \frac{y - 4}{-1} = \frac{z + 1}{1} \quad \text{and} \quad \frac{x - 5}{4} = \frac{y - 1}{3} = \frac{z - 1}{2}
\]
respectively.

(i) Find the equation of the plane \( \Pi_1 \) which contains \( l_1 \) and is parallel to \( l_2 \), giving your answer in the form \( \mathbf{r} \cdot \mathbf{n} = p \).

(ii) Find the equation of the plane \( \Pi_2 \) which contains \( l_2 \) and is parallel to \( l_1 \), giving your answer in the form \( \mathbf{r} \cdot \mathbf{n} = p \).

(iii) Find the distance between the planes \( \Pi_1 \) and \( \Pi_2 \).

(iv) State the relationship between the answer to part (iii) and the lines \( l_1 \) and \( l_2 \).

Q3, (Jun 2008, Q2)
Find the acute angle between the line with equation \( \mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \) and the plane with equation \( \mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \).

Q4, (Jun 2008, Q5)
Two lines have equations
\[
\frac{x - k}{2} = \frac{y + 1}{-5} = \frac{z - 1}{-3} \quad \text{and} \quad \frac{x - k}{1} = \frac{y + 4}{-4} = \frac{z}{-2},
\]
where \( k \) is a constant.

(i) Show that, for all values of \( k \), the lines intersect, and find their point of intersection in terms of \( k \).

(ii) For the case \( k = 1 \), find the equation of the plane in which the lines lie, giving your answer in the form \( ax + by + cz = d \).

Q5, (Jun 2009, Q3)
A line \( l \) has equation \( \frac{x - 6}{-4} = \frac{y + 7}{8} = \frac{z + 10}{7} \) and a plane \( p \) has equation \( 3x - 4y - 2z = 8 \).

(i) Find the point of intersection of \( l \) and \( p \).

(ii) Find the equation of the plane which contains \( l \) and is perpendicular to \( p \), giving your answer in the form \( ax + by + cz = d \).
Q6, (Jan 2011, Q2)

Two intersecting lines, lying in a plane $p$, have equations

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{-3} \quad \text{and} \quad \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-4}{4}.$$ 

(i) Obtain the equation of $p$ in the form $2x - y + z = 3$. [3]

(ii) Plane $q$ has equation $2x - y + z = 21$. Find the distance between $p$ and $q$. [3]

Q7, (Jun 2011, Q1)

A line $l$ has equation $\frac{x-1}{5} = \frac{y-6}{6} = \frac{z+3}{-7}$ and a plane $p$ has equation $x + 2y - z = 40$.

(i) Find the acute angle between $l$ and $p$. [4]

(ii) Find the perpendicular distance from the point $(1, 6, -3)$ to $p$. [2]

Q8, (Jun 2013, Q1)

The plane $\Pi$ passes through the points with coordinates $(1, 6, 2)$, $(5, 2, 1)$ and $(1, 0, -2)$.

(i) Find a vector equation of $\Pi$ in the form $\mathbf{r} = a + \lambda \mathbf{b} + \mu \mathbf{c}$. [2]

(ii) Find a cartesian equation of $\Pi$. [4]

Q9, (Jun 2013, Q6,ii)

The plane $\Pi$ has equation $x + 2y - 2z = 5$. The line $l$ has equation $\frac{x-1}{2} = \frac{y+1}{5} = \frac{z-2}{1}$.

(i) Find the coordinates of the point of intersection of $l$ with the plane $\Pi$. [3]

(ii) Calculate the acute angle between $l$ and $\Pi$. [3]

Q10, (Jun 2015, Q3)

The plane $\Pi$ passes through the points $(1, 2, 1)$, $(2, 3, 6)$ and $(4, -1, 2)$.

(i) Find a cartesian equation of the plane $\Pi$. [5]

The line $l$ has equation $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$.

(ii) Find the coordinates of the point of intersection of $\Pi$ and $l$. [3]

(iii) Find the acute angle between $\Pi$ and $l$. [3]