Lines and Planes (Equations, Angles Between and Intersection) (From OCR 4727)

Q1, (Jun 2007, Q2)

A line l has equation $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ and a plane Π has equation $8x - 7y + 10z = 7$. Determine whether l lies in Π, is parallel to Π without intersecting it, or intersects Π at one point. [5]

Q2, (Jun 2007, Q6)

Lines l_1 and l_2 have equations

$$\frac{x-3}{2} = \frac{y-4}{-1} = \frac{z+1}{1} \quad \text{and} \quad \frac{x-5}{4} = \frac{y-1}{3} = \frac{z-1}{2}$$

respectively.

(i) Find the equation of the plane Π_1 which contains l_1 and is parallel to l_2, giving your answer in the form $\mathbf{r.n} = p$. [5]

(ii) Find the equation of the plane Π_2 which contains l_2 and is parallel to l_1, giving your answer in the form $\mathbf{r.n} = p$. [2]

(iii) Find the distance between the planes Π_1 and Π_2. [2]

(iv) State the relationship between the answer to part **(iii)** and the lines l_1 and l_2. [1]

Q3, (Jun 2008, Q2)

Find the acute angle between the line with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and the plane with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$. [7]

Q4, (Jun 2008, Q5)

Two lines have equations

$$\frac{x-k}{2} = \frac{y+1}{-5} = \frac{z-1}{-3} \quad \text{and} \quad \frac{x-k}{1} = \frac{y+4}{-4} = \frac{z}{-2},$$

where k is a constant.

(i) Show that, for all values of k, the lines intersect, and find their point of intersection in terms of k. [6]

(ii) For the case $k = 1$, find the equation of the plane in which the lines lie, giving your answer in the form $ax + by + cz = d$. [4]

Q5, (Jun 2009, Q3)

A line l has equation $\frac{x-6}{-4} = \frac{y+7}{8} = \frac{z+10}{7}$ and a plane p has equation $3x - 4y - 2z = 8$.

(i) Find the point of intersection of l and p. **[3]**

(ii) Find the equation of the plane which contains l and is perpendicular to p, giving your answer in the form $ax + by + cz = d$. **[5]**

Q6, (Jan 2011, Q2)

Two intersecting lines, lying in a plane p, have equations

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{-3} \quad \text{and} \quad \frac{x-1}{-1}=\frac{y-3}{2}=\frac{z-4}{4}.$$

(i) Obtain the equation of p in the form $2x - y + z = 3$. **[3]**

(ii) Plane q has equation $2x - y + z = 21$. Find the distance between p and q. **[3]**

Q7, (Jun 2011, Q1)

A line l has equation $\frac{x-1}{5}=\frac{y-6}{6}=\frac{z+3}{-7}$ and a plane p has equation $x + 2y - z = 40$.

(i) Find the acute angle between l and p. **[4]**

(ii) Find the perpendicular distance from the point $(1, 6, -3)$ to p. **[2]**

Q8, (Jun 2013, Q1)

The plane Π passes through the points with coordinates $(1, 6, 2)$, $(5, 2, 1)$ and $(1, 0, -2)$.

(i) Find a vector equation of Π in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. **[2]**

(ii) Find a cartesian equation of Π. **[4]**

Q9, (Jun 2013, Q6i,ii)

The plane Π has equation $x + 2y - 2z = 5$. The line l has equation $\frac{x-1}{2}=\frac{y+1}{5}=\frac{z-2}{1}$.

(i) Find the coordinates of the point of intersection of l with the plane Π. **[3]**

(ii) Calculate the acute angle between l and Π. **[3]**

Q10, (Jun 2015, Q3)

The plane Π passes through the points $(1, 2, 1)$, $(2, 3, 6)$ and $(4, -1, 2)$.

(i) Find a cartesian equation of the plane Π. **[5]**

The line l has equation $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$.

(ii) Find the coordinates of the point of intersection of Π and l. **[3]**

(iii) Find the acute angle between Π and l. **[3]**
