Integration of Trigonometric and Hyperbolic Functions Exam Questions (From OCR 4726)

Q1, (Jan 2007, Q4)

(i) On separate diagrams, sketch the graphs of $y = \sinh x$ and $y = \operatorname{cosech} x$. [3]

(ii) Show that $\operatorname{cosech} x = \frac{2e^x}{e^{2x} - 1}$, and hence, using the substitution $u = e^x$, find $\int \operatorname{cosech} x \, dx$. [6]

Q2, (Jan 2008, Q9)

(i) Prove that $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$. [3]

(ii) Hence, or otherwise, find $\int \frac{1}{\sqrt{4x^2 - 1}} \, dx$. [2]

(iii) By means of a suitable substitution, find $\int \sqrt{4x^2 - 1} \, dx$. [6]

Q3, (Jan 2009, Q4)

(i) By means of a suitable substitution, show that

$$\int \frac{x^2}{\sqrt{x^2 - 1}} \, dx$$

can be transformed to $\int \cosh^2 \theta \, d\theta$. **[2]**

(ii) Hence show that $\int \frac{x^2}{\sqrt{x^2 - 1}} \, dx = \frac{1}{2}x\sqrt{x^2 - 1} + \frac{1}{2}\cosh^{-1} x + c$. **[4]**

Q4, (Jun 2009, Q6)

Given that

$$\int_0^1 \frac{1}{\sqrt{16 + 9x^2}} \, dx + \int_0^2 \frac{1}{\sqrt{9 + 4x^2}} \, dx = \ln a,$$

find the exact value of a. **[6]**

Q5, (Jun 2011, Q7)

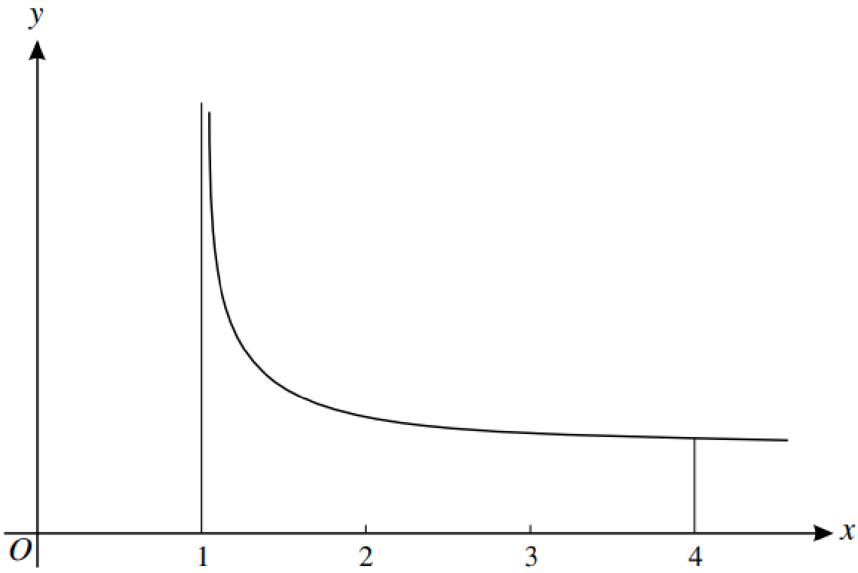
(i) Sketch the graph of $y = \tanh x$ and state the value of the gradient when $x = 0$. On the same axes, sketch the graph of $y = \tanh^{-1} x$. Label each curve and give the equations of the asymptotes. **[4]**

(ii) Find $\int_0^k \tanh x \, dx$, where $k > 0$. **[2]**

(iii) Deduce, or show otherwise, that $\int_0^{\tanh k} \tanh^{-1} x \, dx = k \tanh k - \ln(\cosh k)$. **[4]**

Q6, (Jun 2011, Q8)

(i) Use the substitution $x = \cosh^2 u$ to find $\int \sqrt{\frac{x}{x-1}}\, dx$, giving your answer in the form $f(x) + \ln(g(x))$. **[7]**



(ii) Hence calculate the exact area of the region between the curve $y = \sqrt{\frac{x}{x-1}}$, the x-axis and the lines $x = 1$ and $x = 4$ (see diagram). **[1]**

(iii) What can you say about the volume of the solid of revolution obtained when the region defined in part **(ii)** is rotated completely about the x-axis? Justify your answer. **[3]**

Q7, (Jan 2012, Q2)

By first completing the square in the denominator, find the exact value of

$$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2 - 4x + 5}\, dx .$$

[5]

Q8, (Jun 2012, Q1)

Express $\operatorname{sech} 2x$ in terms of exponentials and hence, by using the substitution $u = e^{2x}$, find $\int \operatorname{sech} 2x\, dx$. **[5]**

Q9, (Jan 2013, Q6)

By first completing the square, find $\int_0^1 \frac{1}{\sqrt{x^2 + 4x + 8}}\, dx$, giving your answer in an exact logarithmic form. **[6]**

Q10, (Jun 2014, Q1)

Find $\int_0^2 \frac{1}{\sqrt{4 + x^2}}\, dx$, giving your answer exactly in logarithmic form. **[3]**

Q11, (Jun 2015, Q3)

By first completing the square, find the exact value of $\int_{\frac{1}{2}}^{1} \frac{1}{\sqrt{2x-x^2}}\,dx$. **[5]**

Q12, (Jun 2017, Q2)

By first completing the square in the denominator, find the exact value of

$$\int_{3}^{4} \frac{1}{x^2-6x+10}\,dx.$$

[4]
