First Order Differential Equations Involving Substitutions (From OCR 4727)

Q1, (Jun 2008, Q3)

(i) Use the substitution $z = x + y$ to show that the differential equation

$$\frac{dy}{dx} = \frac{x+y+3}{x+y-1} \qquad \text{(A)}$$

may be written in the form $\frac{dz}{dx} = \frac{2(z+1)}{z-1}$. [3]

(ii) Hence find the general solution of the differential equation (A). [4]

Q2, (Jan 2009, Q5)

The variables x and y are related by the differential equation

$$x^3\frac{dy}{dx} = xy + x + 1. \qquad \text{(A)}$$

(i) Use the substitution $y = u - \frac{1}{x}$, where u is a function of x, to show that the differential equation may be written as

$$x^2\frac{du}{dx} = u.$$ **[4]**

(ii) Hence find the general solution of the differential equation (A), giving your answer in the form $y = f(x)$. **[5]**

Q3, (Jun 2010, Q4) Q7, (Jun 2010, Q4)

In this question you may use the result that $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$

(i) Use the substitution $y = xz$ to find the general solution of the differential equation

$$x\frac{dy}{dx} - y = x\cos\left(\frac{y}{x}\right),$$

giving your answer in a form without logarithms. **[6]**

(ii) Find the solution of the differential equation for which $y = \pi$ when $x = 4$. **[2]**

Q4, (Jun 2011, Q5)

The substitution $y = u^k$, where k is an integer, is to be used to solve the differential equation

$$x\frac{dy}{dx} + 3y = x^2y^2 \qquad \text{(A)}$$

by changing it into an equation (B) in the variables u and x.

(i) Show that equation (B) may be written in the form

$$\frac{du}{dx} + \frac{3}{kx}u = \frac{1}{k}xu^{k+1}.$$ **[4]**

(ii) Write down the value of k for which the integrating factor method may be used to solve equation (B). **[1]**

(iii) Using this value of k, solve equation (B) and hence find the general solution of equation (A), giving your answer in the form $y = f(x)$. **[4]**

Q5, (Jan 2012, Q1)

The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{2x^2 + y^2}{xy}. \qquad \text{(A)}$$

(i) Use the substitution $y = ux$, where u is a function of x, to obtain the differential equation

$$x\frac{du}{dx} = \frac{2}{u}.$$ **[3]**

(ii) Hence find the general solution of the differential equation (A), giving your answer in the form $y^2 = f(x)$. **[4]**

Q6, (Jun 2013, Q3)

The differential equation

$$3xy^2\frac{dy}{dx} + 2y^3 = \frac{\cos x}{x}$$

is to be solved for $x > 0$. Use the substitution $u = y^3$ to find the general solution for y in terms of x. **[8]**

Q7, (Jun 2014, Q2)

Use the substitution $u = y^2$ to find the general solution of the differential equation

$$\frac{dy}{dx} - 2y = \frac{e^x}{y}$$

for y in terms of x. **[8]**

Q8, (Jun 2016, Q3)

The differential equation

$$\frac{2}{y} - \frac{x}{y^2}\frac{dy}{dx} = \frac{1}{x^2}$$

is to be solved subject to the condition $y = 1$ when $x = 1$.

(i) Show that $y = \frac{1}{u}$ transforms the differential equation into

$$x\frac{du}{dx} + 2u = \frac{1}{x^2}.$$ **[3]**

(ii) Find y in terms of x. **[7]**
