Q1, (Jun 2007, Q8)

(i) Find the general solution of the differential equation

\[
\frac{dy}{dx} + y \tan x = \cos^3 x,
\]

expressing \(y\) in terms of \(x\) in your answer. \[8\]

(ii) Find the particular solution for which \(y = 2\) when \(x = \pi\). \[2\]

Q2, (Jan 2008, Q5)

(i) Find the general solution of the differential equation

\[
\frac{dy}{dx} + \frac{y}{x} = \sin 2x,
\]

expressing \(y\) in terms of \(x\) in your answer. \[6\]

In a particular case, it is given that \(y = \frac{2}{\pi}\) when \(x = \frac{1}{4}\pi\).

(ii) Find the solution of the differential equation in this case. \[2\]

(iii) Write down a function to which \(y\) approximates when \(x\) is large and positive. \[1\]

Q3, (Jun 2009, Q4) [Modified]

The differential equation

\[
\frac{dy}{dx} + \frac{1}{1-x^2} \ y = (1-x)^{\frac{1}{2}}, \quad \text{where} \ |x| < 1,
\]

can be solved by the integrating factor method.

(i) Use partial fractions or another appropriate method in order to show that the integrating factor can be written as \(\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\). \[2\]

(ii) Hence find the solution of the differential equation for which \(y = 2\) when \(x = 0\), giving your answer in the form \(y = f(x)\). \[6\]

Q4, (Jan 2010, Q3)

Use the integrating factor method to find the solution of the differential equation

\[
\frac{dy}{dx} + 2y = e^{-3x}
\]

for which \(y = 1\) when \(x = 0\). Express your answer in the form \(y = f(x)\). \[6\]

Q5, (Jan 2011, Q1)

(i) Find the general solution of the differential equation

\[
\frac{dy}{dx} + xy = x e^{\frac{1}{2}x^2},
\]

giving your answer in the form \(y = f(x)\). \[4\]

(ii) Find the particular solution for which \(y = 1\) when \(x = 0\). \[2\]
Q6, (Jun 2011, Q3)
NOTE: This question requires methods used in Second-Order Differential Equations

The variables $x$ and $y$ satisfy the differential equation

$$\frac{dy}{dx} + 4y = 5 \cos 3x.$$  

(i) Find the complementary function. [2]

(ii) Hence, or otherwise, find the general solution. [7]

(iii) Find the approximate range of values of $y$ when $x$ is large and positive. [2]

Q7, (Jun 2012, Q3)
Find the solution of the differential equation

$$\frac{dy}{dx} + y \cot x = 2x$$

for which $y = 2$ when $x = \frac{1}{6}\pi$. Give your answer in the form $y = f(x)$. [9]

Q8, (Jan 2013, Q3)
Solve the differential equation $x \frac{dy}{dx} - 3y = x^4 e^{2x}$ for $y$ in terms of $x$, given that $y = 0$ when $x = 1$. [8]

Q9, (Jun 2013, Q3)
The differential equation

$$3xy^2 \frac{dy}{dx} + 2y^3 = \frac{\cos x}{x}$$
is to be solved for $x > 0$. Use the substitution $u = y^3$ to find the general solution for $y$ in terms of $x$. [8]

Q10, (Jun 2014, Q2)
Use the substitution $u = y^2$ to find the general solution of the differential equation

$$\frac{dy}{dx} - 2y = \frac{e^x}{y}$$

for $y$ in terms of $x$. [8]

Q11, (Jun 2015, Q5)
Find the particular solution of the differential equation

$$x \frac{dy}{dx} + 3y = x^2 + x$$

for which $y = 1$ when $x = 1$, giving $y$ in terms of $x$. [8]

Q12, (Jun 2017, Q1)
Solve the differential equation

$$\frac{dy}{dx} + y \cot x = 9 \cosec x$$

to find $y$ in terms of $x$ subject to the condition $y = \pi$ when $x = \frac{1}{6}\pi$. [8]