Q1, (Jun 2006, Q2)

(i) Given that \( y = \tan^{-1} x \), prove that \( \frac{dy}{dx} = \frac{1}{1 + x^2} \). [3]

(ii) Verify that \( y = \tan^{-1} x \) satisfies the equation

\[
(1 + x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0.
\] [3]

Q2, (Jun 2007, Q4)

(i) Given that

\[ y = x\sqrt{1 - x^2} - \cos^{-1} x, \]

find \( \frac{dy}{dx} \) in a simplified form. [4]

(ii) Hence, or otherwise, find the exact value of \( \int_{0}^{1} 2\sqrt{1 - x^2} \, dx \). [3]

Q3, (Jan 2008, Q9i)

(i) Prove that \( \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}} \). [3]

Q4, (Jan 2010, Q9)

(i) Given that \( y = \tanh^{-1} x \), for \(-1 < x < 1\), prove that \( y = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \). [3]

(ii) It is given that \( f(x) = a \cosh x - b \sinh x \), where \( a \) and \( b \) are positive constants.

(a) Given that \( b \geq a \), show that the curve with equation \( y = f(x) \) has no stationary points. [3]

(b) In the case where \( a > 1 \) and \( b = 1 \), show that \( f(x) \) has a minimum value of \( \sqrt{a^2 - 1} \). [6]

Q5, (Jun 2010, Q6)

(i) Show that \( \frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2 + 1}} \). [2]

(ii) Given that \( y = \cosh(a \sinh^{-1} x) \), where \( a \) is a constant, show that

\[
(x^2 + 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - a^2 y = 0.
\] [5]

Q6, (Jun 2011, Q5)

(i) Prove that, if \( y = \sin^{-1} x \), then \( \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}} \). [3]

(ii) Find the Maclaurin series for \( \sin^{-1} x \), up to and including the term in \( x^3 \). [5]

(iii) Use the result of part (ii) and the Maclaurin series for \( \ln(1 + x) \) to find the Maclaurin series for \( (\sin^{-1} x) \ln(1 + x) \), up to and including the term in \( x^4 \). [4]
Q7, (Jan 2012, Q6)

(i) Prove that the derivative of $\cos^{-1} x$ is $-\frac{1}{\sqrt{1-x^2}}$. [3]

A curve has equation $y = \cos^{-1} (1-x^2)$, for $0 < x < \sqrt{2}$.

(ii) Find and simplify $\frac{dy}{dx}$, and hence show that

$$\left(2-x^2\right)\frac{d^2y}{dx^2} = x \frac{dy}{dx}.$$ [5]

Q8, (Jun 2013, Q3)

It is given that $f(x) = \tanh^{-1}\left(\frac{1-x}{3+x}\right)$ for $x > -1$.

(i) Show that $f''(x) = \frac{1}{2(x+1)^2}$. [6]

(ii) Hence find the Maclaurin series for $f(x)$ up to and including the term in $x^2$. [4]

Q9, (Jun 2014, Q4)

The curves $y = \cos^{-1} x$ and $y = \tan^{-1}(\sqrt{2}x)$ intersect at a point $A$.

(i) Verify that the coordinates of $A$ are $(\frac{1}{\sqrt{2}}, \frac{1}{4} \pi)$. [2]

(ii) Determine whether the tangents to the curves at $A$ are perpendicular. [4]

Q10, (Jun 2014, Q6)

(i) Given that $y = \cosh^{-1} x$, show that $y = \ln(x + \sqrt{x^2 - 1})$. [4]

(ii) Show that $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$. [2]

(iii) Solve the equation $\cosh x = 3$, giving your answers in logarithmic form. [3]

Q11, (Jun 2015, Q5)

It is given that $y = \sin^{-1} 2x$.

(i) Using the derivative of $\sin^{-1} x$ given in the List of Formulae, find $\frac{dy}{dx}$. [1]

(ii) Show that $(1-4x^2)\frac{d^2y}{dx^2} = 4x\frac{dy}{dx}$. [3]

(iii) Hence show that $(1-4x^2)\frac{d^3y}{dx^3} - 12x\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = 0$. [2]

(iv) Using your results from parts (i), (ii) and (iii), find the Maclaurin series for $\sin^{-1}2x$ up to and including the term in $x^3$. [3]