Applied First Order Differential Equation (From OCR 4730)

Q1, (Jan 2007, Q5)

The pilot of a hot air balloon keeps it at a fixed altitude by dropping sand from the balloon. Each grain of sand has mass m kg and is released from rest. When a grain has fallen a distance x m, it has speed $v\ \mathrm{m\,s^{-1}}$. Each grain falls vertically and the only forces acting on it are its weight and air resistance of magnitude mkv^2 N, where k is a positive constant.

(i) Show that $\left(\dfrac{v}{g - kv^2}\right)\dfrac{\mathrm{d}v}{\mathrm{d}x} = 1$. [2]

(ii) Find v^2 in terms of k, g and x. Hence show that, as x becomes large, the limiting value of v is $\sqrt{\dfrac{g}{k}}$. [7]

(iii) Given that the altitude of the balloon is 300 m and that each grain strikes the ground at 90% of its limiting velocity, find k. [3]

Q2, (Jun 2007, Q3)

A particle P of mass 0.2 kg is projected horizontally with speed $u\ \mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface. P moves in a straight line and, at time t s after projection, P has speed $v\ \mathrm{m\,s^{-1}}$ and is x m from O. The only force acting on P has magnitude $0.4v^2$ N and is directed towards O.

(i) Show that $\dfrac{1}{v}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -2$. [2]

(ii) Hence show that $v = u\mathrm{e}^{-2x}$. [4]

(iii) Find u, given that $x = 2$ when $t = 4$. [4]

Q3, (Jan 2009, Q6)

A stone of mass 0.125 kg falls freely under gravity, from rest, until it has travelled a distance of 10 m. The stone then continues to fall in a medium which exerts an upward resisting force of $0.025v$ N, where $v\ \mathrm{m\,s^{-1}}$ is the speed of the stone t s after the instant that it enters the resisting medium.

(i) Show by integration that $v = 49 - 35\mathrm{e}^{-0.2t}$. **[8]**

(ii) Find how far the stone travels during the first 3 seconds in the medium. **[4]**

Q4, (Jun 2009, Q4)

A motor-cycle, whose mass including the rider is 120 kg, is decelerating on a horizontal straight road. The motor-cycle passes a point A with speed $40\ \mathrm{m\,s^{-1}}$ and when it has travelled a distance of x m beyond A its speed is $v\ \mathrm{m\,s^{-1}}$. The engine develops a constant power of 8 kW and resistances are modelled by a force of $0.25v^2$ N opposing the motion.

(i) Show that $\dfrac{480v^2}{v^3 - 32\,000}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -1$. **[5]**

(ii) Find the speed of the motor-cycle when it has travelled 500 m beyond A. **[6]**

Q5, (Jun 2011, Q3)

A particle P of mass 0.25 kg is projected horizontally with speed $5\,\mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface and moves in a straight line on the surface. The only horizontal force acting on P has magnitude $0.2v^2$ N, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P at time t s after it is projected from O. This force is directed towards O.

(i) Find an expression for v in terms of t. **[5]**

The particle P passes through a point X with speed $0.2\,\mathrm{m\,s^{-1}}$.

(ii) Find the average speed of P for its motion between O and X. **[5]**

Q6, (Jan 2013, Q3)

At time $t = 0$ s a particle P, of mass 0.3 kg, is 1 m away from a point O on a smooth horizontal plane and is moving away from O with speed $\sqrt{5}\,\mathrm{m\,s^{-1}}$. The only horizontal force acting on P has magnitude $1.5x$ N, where x is the distance OP, and acts away from O.

(i) Show that the speed of P, $v\,\mathrm{m\,s^{-1}}$, is given by $v = \sqrt{5}x$. **[4]**

(ii) Find an expression for v in terms of t. **[4]**

Q7, (Jun 2014, Q4)

A particle P of mass 0.4 kg is projected horizontally with speed $2\,\mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface. At time t s after projection P is x m from O and is moving away from O with speed $v\,\mathrm{m\,s^{-1}}$. There is a force of magnitude $1.6v^2$ N resisting the motion of P.

(i) Find an expression for $\frac{\mathrm{d}v}{\mathrm{d}x}$ in terms of v, and hence show that $v = 2e^{-4x}$. **[5]**

(ii) Find the distance travelled by P in the 0.5 seconds after it leaves O. **[5]**

Q8, (Jun 2015, Q4)

A particle of mass 0.4 kg, moving on a smooth horizontal surface, passes through a point O with velocity $10\,\mathrm{m\,s^{-1}}$. At time t s after the particle passes through O, the particle has a displacement x m from O, has a velocity $v\,\mathrm{m\,s^{-1}}$ away from O, and is acted on by a force of magnitude $\frac{1}{8}v$ N acting towards O. Find

(i) the time taken for the velocity of the particle to reduce from $10\,\mathrm{m\,s^{-1}}$ to $5\,\mathrm{m\,s^{-1}}$, **[5]**

(ii) the average velocity of the particle over this time. **[6]**

Q9, (Jun 2016, Q2)

A particle Q of mass 0.2 kg is projected horizontally with velocity $4\,\mathrm{m\,s^{-1}}$ from a fixed point A on a smooth horizontal surface. At time t s after projection Q is x m from A and is moving away from A with velocity $v\,\mathrm{m\,s^{-1}}$. There is a force of $3\cos 2t$ N acting on Q in the positive x-direction.

(i) Find an expression for the velocity of Q at time t. State the maximum and minimum values of the velocity of Q as t varies. **[4]**

(ii) Find the average velocity of Q between times $t = \pi$ and $t = \frac{3}{2}\pi$. **[4]**
