Problem Solving With Circular Motion

Q1. (OCR 4729, Jun 2013, Q5)

A vertical hollow cylinder of radius 0.4 m is rotating about its axis. A particle \( P \) is in contact with the rough inner surface of the cylinder. The cylinder and \( P \) rotate with the same constant angular speed. The coefficient of friction between \( P \) and the cylinder is \( \mu \).

(i) Given that the angular speed of the cylinder is \( 7 \text{ rad s}^{-1} \) and \( P \) is on the point of moving downwards, find the value of \( \mu \). \([5]\)

The particle is now attached to one end of a light inextensible string of length 0.5 m. The other end is fixed to a point \( A \) on the axis of the cylinder (see diagram).

(ii) Find the angular speed for which the contact force between \( P \) and the cylinder becomes zero. \([5]\)

Q2. (OCR 4729, Jun 2015, Q8)

Two small spheres, \( A \) and \( B \), are free to move on the inside of a smooth hollow cylinder, in such a way that they remain in contact with both the curved surface of the cylinder and its horizontal base. The mass of \( A \) is 0.4 kg, the mass of \( B \) is 0.5 kg and the radius of the cylinder is 0.6 m (see diagram). The coefficient of restitution between \( A \) and \( B \) is 0.35. Initially, \( A \) and \( B \) are at opposite ends of a diameter of the base of the cylinder with \( A \) travelling at a constant speed of \( v \text{ m s}^{-1} \) and \( B \) stationary. The magnitude of the force exerted on \( A \) by the curved surface of the cylinder is 6 N.

(i) Show that \( v = 3 \). \([2]\)

(ii) Calculate the speeds of the particles after \( A \)'s first impact with \( B \). \([6]\)

Sphere \( B \) is removed from the cylinder and sphere \( A \) is now set in motion with constant angular speed \( \omega \text{ rad s}^{-1} \). The magnitude of the total force exerted on \( A \) by the cylinder is 4.9 N.

(iii) Find \( \omega \). \([4]\)
A conical shell has radius 6 m and height 8 m. The shell, with its vertex \( V \) downwards, is rotating about its vertical axis. A particle, of mass 0.4 kg, is in contact with the rough inner surface of the shell. The particle is 4 m above the level of \( V \) (see diagram). The particle and shell rotate with the same constant angular speed. The coefficient of friction between the particle and the shell is \( \mu \).

(i) The frictional force on the particle is \( F_N \) and the normal force of the shell on the particle is \( R_N \). It is given that the speed of the particle is 4.5 m s\(^{-1}\), which is the smallest possible speed for the particle not to slip.

(a) By resolving vertically, show that \( 4F + 3R = 19.6 \). [2]

(b) By finding another equation connecting \( F \) and \( R \), find the values of \( F \) and \( R \) and show that \( \mu = 0.336 \), correct to 3 significant figures. [6]

(ii) Find the largest possible angular speed of the shell for which the particle does not slip. [6]