

**Linear Motion With Variable Acceleration (From OCR 4730)**

**Q1, (Jan 2007, Q5)**

The pilot of a hot air balloon keeps it at a fixed altitude by dropping sand from the balloon. Each grain of sand has mass  $m$  kg and is released from rest. When a grain has fallen a distance  $x$  m, it has speed  $v$  m s<sup>-1</sup>. Each grain falls vertically and the only forces acting on it are its weight and air resistance of magnitude  $mkv^2$  N, where  $k$  is a positive constant.

(i) Show that  $\left(\frac{v}{g - kv^2}\right) \frac{dv}{dx} = 1$ . [2]

(ii) Find  $v^2$  in terms of  $k$ ,  $g$  and  $x$ . Hence show that, as  $x$  becomes large, the limiting value of  $v$  is  $\sqrt{\frac{g}{k}}$ . [7]

(iii) Given that the altitude of the balloon is 300 m and that each grain strikes the ground at 90% of its limiting velocity, find  $k$ . [3]

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**Q2, (Jun 2007, Q3)**

A particle  $P$  of mass 0.2 kg is projected horizontally with speed  $u$  m s<sup>-1</sup> from a fixed point  $O$  on a smooth horizontal surface.  $P$  moves in a straight line and, at time  $t$  s after projection,  $P$  has speed  $v$  m s<sup>-1</sup> and is  $x$  m from  $O$ . The only force acting on  $P$  has magnitude  $0.4v^2$  N and is directed towards  $O$ .

(i) Show that  $\frac{1}{v} \frac{dv}{dx} = -2$ . [2]

(ii) Hence show that  $v = ue^{-2x}$ . [4]

(iii) Find  $u$ , given that  $x = 2$  when  $t = 4$ . [4]

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**Q3, (Jan 2009, Q6)**

A stone of mass 0.125 kg falls freely under gravity, from rest, until it has travelled a distance of 10 m. The stone then continues to fall in a medium which exerts an upward resisting force of  $0.025v$  N, where  $v$  m s<sup>-1</sup> is the speed of the stone  $t$  s after the instant that it enters the resisting medium.

(i) Show by integration that  $v = 49 - 35e^{-0.2t}$ . [8]

(ii) Find how far the stone travels during the first 3 seconds in the medium. [4]

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**Q4, (Jun 2009, Q4)**

A motor-cycle, whose mass including the rider is 120 kg, is decelerating on a horizontal straight road. The motor-cycle passes a point  $A$  with speed 40 m s<sup>-1</sup> and when it has travelled a distance of  $x$  m beyond  $A$  its speed is  $v$  m s<sup>-1</sup>. The engine develops a constant power of 8 kW and resistances are modelled by a force of  $0.25v^2$  N opposing the motion.

(i) Show that  $\frac{480v^2}{v^3 - 32000} \frac{dv}{dx} = -1$ . [5]

(ii) Find the speed of the motor-cycle when it has travelled 500 m beyond  $A$ . [6]

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**Q5, (Jun 2011, Q3)**

A particle  $P$  of mass  $0.25 \text{ kg}$  is projected horizontally with speed  $5 \text{ m s}^{-1}$  from a fixed point  $O$  on a smooth horizontal surface and moves in a straight line on the surface. The only horizontal force acting on  $P$  has magnitude  $0.2v^2 \text{ N}$ , where  $v \text{ m s}^{-1}$  is the velocity of  $P$  at time  $t \text{ s}$  after it is projected from  $O$ . This force is directed towards  $O$ .

(i) Find an expression for  $v$  in terms of  $t$ . [5]

The particle  $P$  passes through a point  $X$  with speed  $0.2 \text{ m s}^{-1}$ .

(ii) Find the average speed of  $P$  for its motion between  $O$  and  $X$ . [5]

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**Q6, (Jan 2013, Q3)**

At time  $t = 0 \text{ s}$  a particle  $P$ , of mass  $0.3 \text{ kg}$ , is  $1 \text{ m}$  away from a point  $O$  on a smooth horizontal plane and is moving away from  $O$  with speed  $\sqrt{5} \text{ m s}^{-1}$ . The only horizontal force acting on  $P$  has magnitude  $1.5x \text{ N}$ , where  $x$  is the distance  $OP$ , and acts away from  $O$ .

(i) Show that the speed of  $P$ ,  $v \text{ m s}^{-1}$ , is given by  $v = \sqrt{5}x$ . [4]

(ii) Find an expression for  $v$  in terms of  $t$ . [4]

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**Q7, (Jun 2014, Q4)**

A particle  $P$  of mass  $0.4 \text{ kg}$  is projected horizontally with speed  $2 \text{ m s}^{-1}$  from a fixed point  $O$  on a smooth horizontal surface. At time  $t \text{ s}$  after projection  $P$  is  $x \text{ m}$  from  $O$  and is moving away from  $O$  with speed  $v \text{ m s}^{-1}$ . There is a force of magnitude  $1.6v^2 \text{ N}$  resisting the motion of  $P$ .

(i) Find an expression for  $\frac{dv}{dx}$  in terms of  $v$ , and hence show that  $v = 2e^{-4x}$ . [5]

(ii) Find the distance travelled by  $P$  in the  $0.5$  seconds after it leaves  $O$ . [5]

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**Q8, (Jun 2015, Q4)**

A particle of mass  $0.4 \text{ kg}$ , moving on a smooth horizontal surface, passes through a point  $O$  with velocity  $10 \text{ m s}^{-1}$ . At time  $t \text{ s}$  after the particle passes through  $O$ , the particle has a displacement  $x \text{ m}$  from  $O$ , has a velocity  $v \text{ m s}^{-1}$  away from  $O$ , and is acted on by a force of magnitude  $\frac{1}{8}v \text{ N}$  acting towards  $O$ . Find

(i) the time taken for the velocity of the particle to reduce from  $10 \text{ m s}^{-1}$  to  $5 \text{ m s}^{-1}$ , [5]

(ii) the average velocity of the particle over this time. [6]

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**Q9, (Jun 2016, Q2)**

A particle  $Q$  of mass  $0.2 \text{ kg}$  is projected horizontally with velocity  $4 \text{ m s}^{-1}$  from a fixed point  $A$  on a smooth horizontal surface. At time  $t \text{ s}$  after projection  $Q$  is  $x \text{ m}$  from  $A$  and is moving away from  $A$  with velocity  $v \text{ m s}^{-1}$ . There is a force of  $3 \cos 2t \text{ N}$  acting on  $Q$  in the positive  $x$ -direction.

(i) Find an expression for the velocity of  $Q$  at time  $t$ . State the maximum and minimum values of the velocity of  $Q$  as  $t$  varies. [4]

(ii) Find the average velocity of  $Q$  between times  $t = \pi$  and  $t = \frac{3}{2}\pi$ . [4]

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