Centres of Mass Involving Integration (From OCR 4763)

Q1, (Jan 2006, Q4)

(i)	$\int y\,dx = \int_0^2 (4-x^2)\,dx \;= \left[4x - \frac{1}{3}x^3\right]_0^2 \quad (=\frac{16}{3})$	B1	
	$\int xy\,dx = \int_0^2 x(4-x^2)\,dx$	M1	
	$= \left[2x^2 - \frac{1}{4}x^4\right]_0^2 \quad (=4)$	A1	
	$\bar{x} = \frac{4}{\frac{16}{3}}$	M1	
	$= 0.75$	E1	Correctly obtained
	$\int \frac{1}{2}y^2\,dx = \int_0^2 \frac{1}{2}(16 - 8x^2 + x^4)\,dx$	M1	
	$= \left[8x - \frac{4}{3}x^3 + \frac{1}{10}x^5\right]_0^2 \quad (=\frac{128}{15})$	A1	
	OR $\int yx\,dy = \int_0^4 y\sqrt{4-y}\,dy$ M1		Valid method of integration
	$= \left[-\frac{2}{3}y(4-y)^{\frac{3}{2}} - \frac{4}{15}(4-y)^{\frac{5}{2}}\right]_0^4$ A1		or $\left[-\frac{8}{3}(4-y)^{\frac{3}{2}} + \frac{2}{5}(4-y)^{\frac{5}{2}}\right]_0^4$
	$\bar{y} = \frac{\frac{128}{15}}{\frac{16}{3}}$	M1	
	$= 1.6$	E1	Correctly obtained
		9	*SR* If $\frac{1}{2}$ is omitted, marks for $\bar{y}$ are M1A0M0E0
(ii)	$\bar{x} = \frac{12\times 0 + 6.5\times 0.75 + 6.5\times 2.75}{12 + 6.5 + 6.5}$	M1 M1	For $6.5\times 0.75 + 6.5\times 2.75$ Using $(\sum m)\bar{x} = \sum mx$
	$= \frac{22.75}{25} = 0.91$	A1	
	$\bar{y} = \frac{12\times 0 + 6.5\times 1.6 + 6.5\times 1.6}{25}$	M1	Using $(\sum m)\bar{y} = \sum my$
	$= \frac{20.8}{25} = 0.832$	A1	
		5	
(iii)		M1 M1	For CM vertically below A For trig in a triangle containing θ, or finding the gradient of AG
	$\tan\theta = \frac{2 - 0.91}{4 - 0.832} \quad (=\frac{1.09}{3.168})$	A1	Correct expression for $\tan\theta$ or $\tan(90-\theta)$
	$\theta = 19.0°$	A1	Accept 0.33 rad
		4	

Q2, (June 2006, Q4)

(i)	$\int \pi y^2 \, dx = \int_1^4 \pi x \, dx$	M1	*π may be omitted throughout*
	$= \left[\frac{1}{2}\pi x^2\right]_1^4 = 7.5\pi$	A1	
	$\int \pi x y^2 \, dx$	M1	
	$= \int_1^4 \pi x^2 \, dx \quad = \left[\frac{1}{3}\pi x^3\right]_1^4 \quad (= 21\pi)$	A1	
	$\bar{x} = \frac{21\pi}{7.5\pi}$	M1	
	$= 2.8$	A1 **6**	
(ii)	Cylinder has mass $3\pi\rho$	B1	Or volume 3π
	Cylinder has CM at $x = 2.5$	B1	
	$(4.5\pi\rho)\bar{x} + (3\pi\rho)(2.5) = (7.5\pi\rho)(2.8)$	M1 A1	Relating three CMs (ρ *and / or* π *may be omitted)* or equivalent, e.g. $\bar{x} = \frac{(7.5\pi\rho)(2.8) - (3\pi\rho)(2.5)}{7.5\pi\rho - 3\pi\rho}$
	$\bar{x} = 3$	E1 **5**	Correctly obtained
(iii)(*A*)	Moments about A, $S \times 3 - 96 \times 2 = 0$	M1	Moments equation
	$S = 64 \text{ N}$	A1	
	Vertically, $R + S = 96$	M1	or another moments equation *Dependent on previous M1*
	$R = 32 \text{ N}$	A1 **4**	
(*B*)	Moments about A, $S \times 3 - 96 \times 2 - 6 \times 1.5 = 0$	M1 A1	Moments equation
	Vertically, $R + S = 96 + 6$ $R = 35 \text{ N}, \quad S = 67 \text{ N}$	A1 **3**	Both correct
	OR Add 3 N to each of *R* and *S* M1 $R = 35 \text{ N}, \quad S = 67 \text{ N}$ A2		*Provided* $R \neq S$ Both correct

Q3, (Jan 2007, Q4)

(i)	Area is $\int_1^a \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^a$	M1	
	$= 1 - \frac{1}{a}$	A1	
	$\int x\,y\,dx = \int_1^a \frac{1}{x}\,dx \quad (= \ln a)$	M1	
	$\bar{x} = \dfrac{\int x\,y\,dx}{\int y\,dx}$	M1	
	$= \dfrac{\ln a}{1-\frac{1}{a}} \quad \left(= \dfrac{a\ln a}{a-1}\right)$	A1	
	$\int \frac{1}{2}y^2\,dx = \int_1^a \frac{1}{2x^4}\,dx = \left[-\frac{1}{6x^3}\right]_1^a$ $= \frac{1}{6}\left(1-\frac{1}{a^3}\right)$	M1	Condone omission of $\frac{1}{2}$
	$\bar{y} = \dfrac{\int \frac{1}{2}y^2\,dx}{\int y\,dx}$	M1	($\frac{1}{2}$ needed for this mark)
	$= \dfrac{\frac{1}{6}\left(1-\frac{1}{a^3}\right)}{1-\frac{1}{a}} = \dfrac{a^3-1}{6(a^3-a^2)}$	E1 **8**	
(ii)	When $a = 2,\ \bar{x} = 2\ln 2,\ \bar{y} = \frac{7}{24}$		
	$\tan\theta = \dfrac{\bar{x}-1}{1-\bar{y}}$	M1	CM vertically below A
	$= \dfrac{2\ln 2 - 1}{1-\frac{7}{24}}$	A1	Correct expression for $\tan\theta$ or $\tan(90-\theta)$
	$\theta = 28.6°$	A1 **3**	

Q4, (Jun 2007, Q4)

(a)	Area is $\int_0^2 x^3\,dx = \left[\frac{1}{4}x^4\right]_0^2 = 4$	B1		
	$\int x\,y\,dx = \int_0^2 x^4\,dx$	M1		
	$= \left[\frac{1}{5}x^5\right]_0^2 = 6.4$	A1		
	$\bar{x} = \frac{6.4}{4} = 1.6$	A1		
	$\int \frac{1}{2}y^2\,dx = \int_0^2 \frac{1}{2}x^6\,dx$	M1		Condone omission of $\frac{1}{2}$
	$= \left[\frac{1}{14}x^7\right]_0^2 = \frac{64}{7}$	A1		
	$\bar{y} = \frac{\int \frac{1}{2}y^2\,dx}{\int y\,dx}$	M1		
	$= \frac{\frac{64}{7}}{4} = \frac{16}{7}$	A1	**8**	Accept 2.3 from correct working
(b)(i)	Volume is $\int \pi y^2\,dx = \int_1^2 \pi(4 - x^2)\,dx$	M1		*π may be omitted throughout*
	$= \pi\left[4x - \frac{1}{3}x^3\right]_1^2 = \frac{5}{3}\pi$	A1		For $\frac{5}{3}$
	$\int \pi x y^2\,dx = \int_1^2 \pi x(4 - x^2)\,dx$	M1		
	$= \pi\left[2x^2 - \frac{1}{4}x^4\right]_1^2 = \frac{9}{4}\pi$	A1		For $\frac{9}{4}$
	$\bar{x} = \frac{\int \pi x y^2\,dx}{\int \pi y^2\,dx}$	M1		
	$= \frac{\frac{9}{4}\pi}{\frac{5}{3}\pi} = \frac{27}{20} = 1.35$	E1	**6**	Must be fully correct
(ii)	Height of solid is $h = 2\sqrt{3}$	B1		
	$T\,h = mg \times 0.35$	M1		Taking moments
	$F = T = 0.101mg, \quad R = mg$	F1		
	Least coefficient of friction is $\frac{F}{R} = 0.101$	A1	**4**	Must be fully correct (e.g. A0 if $m = \frac{5}{3}\pi$ is used)

(iii)	Volume is $\int \pi y^2 \, \mathrm{d}x \;= \pi \int_1^a \frac{1}{x^4} \, \mathrm{d}x$	M1	π may be omitted throughout
	$= \pi \left[-\frac{1}{3x^3} \right]_1^a = \frac{\pi}{3}\left(1 - \frac{1}{a^3}\right)$	A1	
	$\int \pi x y^2 \, \mathrm{d}x \;= \pi \int_1^a \frac{1}{x^3} \, \mathrm{d}x = \pi \left[-\frac{1}{2x^2} \right]_1^a$	M1	
	$= \frac{\pi}{2}\left(1 - \frac{1}{a^2}\right)$		
	$\bar{x} = \frac{\int \pi x y^2 \, \mathrm{d}x}{\int \pi y^2 \, \mathrm{d}x}$	M1	
	$= \frac{\frac{\pi}{2}\left(1 - \frac{1}{a^2}\right)}{\frac{\pi}{3}\left(1 - \frac{1}{a^3}\right)} = \frac{3(a^3 - a)}{2(a^3 - 1)}$	A1	Any correct form
	Since $a > 1, \;\; a^3 - a < a^3 - 1$		
	Hence $\bar{x} < \frac{3}{2}$, i.e. $\bar{x} < 1.5$	M1	or $\bar{x} \to 1.5$ as $a \to \infty$
		E1 **7**	Fully convincing argument

Q5, (Jan 2008, Q4)

(i)	$V=\int_1^8 \pi (x^{-\frac{1}{3}})^2\,dx$	M1		π *may be omitted throughout*
	$=\pi\left[3x^{\frac{1}{3}}\right]_1^8=3\pi$	A1		
	$V\bar{x}=\int_1^8 \pi x(x^{-\frac{1}{3}})^2\,dx$	M1		
	$=\pi\left[\tfrac{3}{4}x^{\frac{4}{3}}\right]_1^8=\frac{45}{4}\pi$	A1		
	$\bar{x}=\dfrac{\frac{45}{4}\pi}{3\pi}$	M1		*Dependent on previous M1M1*
	$=\frac{15}{4}=3.75$	A1	6	
(ii)	$A=\int_1^8 x^{-\frac{1}{3}}\,dx$	M1		
	$=\left[\tfrac{3}{2}x^{\frac{2}{3}}\right]_1^8=\frac{9}{2}=4.5$	A1		
	$A\bar{x}=\int_1^8 x(x^{-\frac{1}{3}})\,dx$	M1		
	$=\left[\tfrac{3}{5}x^{\frac{5}{3}}\right]_1^8=\frac{93}{5}=18.6$	A1		
	$\bar{x}=\frac{18.6}{4.5}=\frac{62}{15}\quad(\approx 4.13)$	A1		
	$A\bar{y}=\int_1^8 \tfrac{1}{2}(x^{-\frac{1}{3}})^2\,dx$	M1		If $\frac{1}{2}$ omitted, award M1A0A0
	$=\left[\tfrac{3}{2}x^{\frac{1}{3}}\right]_1^8=\frac{3}{2}=1.5$	A1		
	$\bar{y}=\frac{1.5}{4.5}=\frac{1}{3}$	A1	8	

(iii)	$(1)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+(3.5)\begin{pmatrix}4.5\\ 0.25\end{pmatrix}=(4.5)\begin{pmatrix}62/15\\ 1/3\end{pmatrix}=\begin{pmatrix}18.6\\ 1.5\end{pmatrix}$	M1 M1		Attempt formula for CM of composite body (one coordinate sufficient) Formulae for both coordinates; signs must now be correct, but areas (1 and 3.5) may be wrong.
	$\bar{x}=2.85$ $\bar{y}=0.625$	A1 A1	4	ft only if $1<\bar{x}<8$ ft only if $0.5<\bar{y}<1$ *Other methods:* M1A1 for $\bar{x}$ M1A1 for $\bar{y}$ *(In each case, M1 requires a complete and correct method leading to a numerical value)*

Q6, (Jan 2009, Q4)

(a)	$\int \frac{1}{2}y^2\,dx = \int_{-a}^{a} \frac{1}{2}(a^2 - x^2)\,dx$	M1	For integral of $(a^2 - x^2)$
	$= \left[\frac{1}{2}(a^2x - \frac{1}{3}x^3) \right]_{-a}^{a}$		
	$= \frac{2}{3}a^3$	A1	
	$\overline{y} = \dfrac{\frac{2}{3}a^3}{\frac{1}{2}\pi a^2}$	M1	*Dependent on previous M1*
	$= \dfrac{4a}{3\pi}$	E1 **4**	
(b)(i)			*π may be omitted throughout*
	$V = \int \pi y^2\,dx = \int_0^h \pi(mx)^2\,dx$	M1	For integral of x^2 or use of $V = \frac{1}{3}\pi r^2 h$ and $r = mh$
	$= \left[\frac{1}{3}\pi m^2 x^3 \right]_0^h = \frac{1}{3}\pi m^2 h^3$	A1	
	$\int \pi x y^2\,dx = \int_0^h \pi x(mx)^2\,dx$	M1	For integral of x^3
	$= \left[\frac{1}{4}\pi m^2 x^4 \right]_0^h = \frac{1}{4}\pi m^2 h^4$	A1	
	$\overline{x} = \dfrac{\frac{1}{4}\pi m^2 h^4}{\frac{1}{3}\pi m^2 h^3}$	M1	*Dependent on M1 for integral of x^3*
	$= \frac{3}{4}h$	E1 **6**	
(ii)	$m_1 = \frac{1}{3}\pi \times 0.7^2 \times 2.4\rho = \frac{1}{3}\pi\rho \times 1.176$		
	$\text{VG}_1 = 1.8$		
	$m_2 = \frac{1}{3}\pi \times 0.4^2 \times 1.1\rho = \frac{1}{3}\pi\rho \times 0.176$	B1	For m_1 and m_2 (or volumes)
	$\text{VG}_2 = 1.3 + \frac{3}{4} \times 1.1 = 2.125$	B1	or $\frac{1}{4} \times 1.1$ from base
	$(m_1 - m_2)(\text{VG}) + m_2(\text{VG}_2) = m_1(\text{VG}_1)$	M1	Attempt formula for composite body
	$(\text{VG}) + 0.176 \times 2.125 = 1.176 \times 1.8$	F1	
	Distance (VG) is 1.74 m	A1 **5**	
(iii)	VQG is a right-angle	M1	
	$\text{VQ} = \text{VG}\cos\theta$ where $\tan\theta = \dfrac{0.7}{2.4}$ $(\theta = 16.26°)$	M1	
	$\text{VQ} = 1.7428 \times \dfrac{24}{25}$		
	$= 1.67$ m	A1 **3**	ft is $\text{VG} \times 0.96$

Q7, (Jun 2010, Q3)

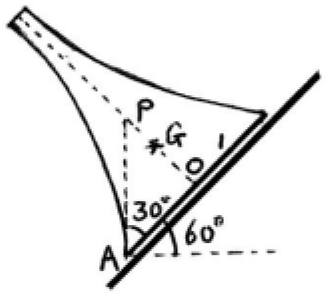
(i)	Volume is $\int_1^5 \pi\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	π may be omitted throughout *Limits not required*
	$=\pi\left[-\frac{1}{x}\right]_1^5 \quad (=\frac{4}{5}\pi)$	A1	For $-\frac{1}{x}$
	$\int \pi x y^2\,\mathrm{d}x = \int_1^5 \pi x\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	*Limits not required*
	$=\pi\left[\ln x\right]_1^5 \quad (=\pi\ln 5)$	A1	For $\ln x$
	$\bar{x}=\frac{\pi\ln 5}{\frac{4}{5}\pi}=\frac{5\ln 5}{4} \quad (2.012)$	A1 **5**	*SR* If exact answers are not seen, deduct only the first A1 affected
(ii)	Area is $\int_1^5 \frac{1}{x}\mathrm{d}x$	M1	*Limits not required*
	$=\left[\ln x\right]_1^5 \quad (=\ln 5)$	A1	For $\ln x$
	$\int x y\,\mathrm{d}x = \int_1^5 x\left(\frac{1}{x}\right)\mathrm{d}x \quad (=\left[x\right]_1^5=4)$	M1	*Limits not required*
	$\bar{x}=\frac{4}{\ln 5} \quad (2.485)$	A1	
	$\int \frac{1}{2}y^2\,\mathrm{d}x = \int_1^5 \frac{1}{2}\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	For $\int\left(\frac{1}{x}\right)^2 \mathrm{d}x$
	$=\left[-\frac{1}{2x}\right]_1^5 \quad (=\frac{2}{5})$	A1	For $-\frac{1}{2x}$
	$\bar{y}=\frac{2/5}{\ln 5}=\frac{2}{5\ln 5} \quad (0.2485)$	A1 **7**	
(iii)	CM of R_2 is $\left(\frac{2}{5\ln 5}, \frac{4}{\ln 5}\right)$	B1B1 ft **2**	*Do not penalise inexact answers in this part*
(iv)		B1	For CM of R_3 is $(\frac{1}{2}, \frac{1}{2})$ (one coordinate is sufficient)
		M1	Using $\sum mx$ with three terms
	$\bar{x}=\frac{(\ln 5)\left(\frac{4}{\ln 5}\right)+(\ln 5)\left(\frac{2}{5\ln 5}\right)+(1)(\frac{1}{2})}{\ln 5+\ln 5+1}$	M1	Using $\frac{\sum mx}{\sum m}$ with at least two terms in each sum
	CM is $\left(\frac{4.9}{2\ln 5+1}, \frac{4.9}{2\ln 5+1}\right) \quad (1.161,\ 1.161)$	A1 cao **4**	

Q8, (Jun 2013, Q4)

(a)	(i)	$V=\int_0^4 \pi x^2(4-x)\,dx$	M1	For $\int\left(x\sqrt{4-x}\right)^2 dx$	π may be omitted throughout
		$=\pi\left[\frac{4}{3}x^3-\frac{1}{4}x^4\right]_0^4 \quad (=\frac{64\pi}{3})$	A1	For $\frac{4}{3}x^3-\frac{1}{4}x^4$	
		$V\bar{x}=\int \pi xy^2\,dx=\int_0^4 \pi x^3(4-x)\,dx$	M1	For $\int xy^2\,dx$	
		$=\pi\left[x^4-\frac{1}{5}x^5\right]_0^4 \quad (=51.2\pi)$	A1	For $x^4-\frac{1}{5}x^5$	
		$\bar{x}=\frac{51.2\pi}{\frac{64}{3}\pi}$	M1	*Dependent on previous M1M1*	
		$=2.4$	A1 **[6]**		
(a)	(ii)		M1	Taking moments	
		$W(2.4\sin\theta)=W(4\cos\theta)$	A1	FT Correct equation for required angle	$W(2.4\cos\phi)=W(4\sin\phi)$ is A0 unless $\theta=90°-\phi$ also appears
		$\tan\theta=\frac{4}{2.4}=\frac{5}{3}$			
		$\theta=59.0°$ (3 sf)	A1 **[3]**	FT is $\tan^{-1}\frac{4}{\bar{x}}$	FT requires $\bar{x}<4$

(b)		$x = 2 + y^{\frac{1}{3}}$	B1		
		$A = \int_0^8 (2 + y^{\frac{1}{3}})\,dy \quad = \left[2y + \frac{3}{4}y^{\frac{4}{3}} \right]_0^8 \quad (= 28)$	B1	FT	Or $32 - \left[\frac{1}{4}(x-2)^4 \right]_2^4$
		$A\bar{x} = \int \frac{1}{2}x^2\,dy = \int_0^8 \frac{1}{2}\left(4 + 4y^{\frac{1}{3}} + y^{\frac{2}{3}} \right) dy$	M1	For $\int x^2\,dy$	Or $32 \times 2 - \int_2^4 xy\,dx$
		$= \left[2y + \frac{3}{2}y^{\frac{4}{3}} + \frac{3}{10}y^{\frac{5}{3}} \right]_0^8 \quad (= 49.6)$	B2	FT for $2y + \frac{3}{2}y^{\frac{4}{3}} + \frac{3}{10}y^{\frac{5}{3}}$ Give B1 for one minor slip in integration, or if ½ omitted	Or $\frac{1}{5}(x-2)^5 + \frac{1}{2}(x-2)^4$ Or $\frac{1}{4}x(x-2)^4 - \frac{1}{20}(x-2)^5$ Or $\frac{1}{5}x^5 - \frac{3}{2}x^4 + 4x^3 - 4x^2$
		$\bar{x} = \frac{49.6}{28} = \frac{62}{35} = 1.77 \quad$ (3 sf)	A1	CAO	*Must be* $\bar{x}$
		$A\bar{y} = \int xy\,dy = \int_0^8 \left(2y + y^{\frac{4}{3}} \right) dy$	M1	For $\int xy\,dy$	Or $32 \times 4 - \int_2^4 (\frac{1}{2})y^2\,dx$
		$= \left[y^2 + \frac{3}{7}y^{\frac{7}{3}} \right]_0^8 \quad (= \frac{832}{7})$	A1	FT for $y^2 + \frac{3}{7}y^{\frac{7}{3}}$	Or B2 for $\frac{1}{14}(x-2)^7$ Give B1 for one minor slip in integration, or if ½ omitted
		$\bar{y} = \frac{\frac{832}{7}}{28} = \frac{208}{49} = 4.24 \quad$ (3 sf)	A1 **[9]**	CAO	*Must be* $\bar{y}$
	OR	Region under curve has CM $(3.6,\ \frac{16}{7})$ $28\bar{x} + 4 \times 3.6 = 32 \times 2$ $\bar{x} = 1.77$ $28\bar{y} + 4 \times \frac{16}{7} = 32 \times 4$ $\bar{y} = 4.24$		B2B2 B1 (for 28) M1 A1 M1 A1	For integrals, as above

Q9, (Jun 2014, Q4)

(i)	Volume is $\int_0^k \pi\left(e^{-x}\right)^2 dx$	M1	*π may be omitted throughout*	
	$= \pi\left[-\frac{1}{2}e^{-2x}\right]_0^k \quad \{= \frac{1}{2}\pi(1-e^{-2k})\}$	A1	For $-\frac{1}{2}e^{-2x}$	
	$\int \pi x y^2 \, dx$	M1		
	$= \int_0^k \pi x e^{-2x} dx = \pi\left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^k$	A1A1	For $-\frac{1}{2}xe^{-2x}$ and $-\frac{1}{4}e^{-2x}$	
	$= \frac{1}{4}\pi(1-2ke^{-2k} - e^{-2k})$			
	$\bar{x} = \dfrac{1-2ke^{-2k} - e^{-2k}}{2(1-e^{-2k})}$	A1	Any correct form	
	$= \dfrac{1-e^{-2k}}{2(1-e^{-2k})} - \dfrac{2ke^{-2k}}{2(1-e^{-2k})} = \dfrac{1}{2} - \dfrac{k}{e^{2k}-1}$	E1		
		[7]		
(ii)	OG $< \frac{1}{2}$ for all values of k	B1	OR $\dfrac{k}{e^{2k}-1} > 0$ o.e. stated or implied *Allow* $\bar{x} \to \frac{1}{2}$ as $k \to \infty$ *for B1*	
		M1	Trigonometry in OAP or OAG	
	$OP = (1)\tan 30° = \dfrac{1}{\sqrt{3}} \quad (= 0.577)$	A1	Or $O\hat{A}G < \tan^{-1}\frac{1}{2} \quad (= 26.6°)$	
	OG < OP (or $O\hat{A}G < 30°$) so G is to the right of AP and solid will not topple	E1	Fully correct explanation	
		[4]		

(iii)		Area is $\int_0^k \mathrm{e}^{-x}\,\mathrm{d}x = \left[-\mathrm{e}^{-x} \right]_0^k \quad (=1-\mathrm{e}^{-k})$	B1		
		$\int xy\,\mathrm{d}x$	M1		
		$= \int_0^k x\mathrm{e}^{-x}\,\mathrm{d}x = \left[-x\mathrm{e}^{-x} - \mathrm{e}^{-x} \right]_0^k$	A1		
		$\bar{x} = \dfrac{1-k\mathrm{e}^{-k}-\mathrm{e}^{-k}}{1-\mathrm{e}^{-k}}$	A1	Any correct form	e.g. $1-\dfrac{k}{\mathrm{e}^k-1}$
		$\int \frac{1}{2}y^2\,\mathrm{d}x$	M1	For $\int \dots y^2\,\mathrm{d}x$	
		$= \int_0^k \frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x = \left[-\frac{1}{4}\mathrm{e}^{-2x} \right]_0^k$	A1		
		$\bar{y} = \dfrac{1-\mathrm{e}^{-2k}}{4(1-\mathrm{e}^{-k})}$	A1	Any correct form	e.g. $\frac{1}{4}(1+\mathrm{e}^{-k})$
			[7]		

(Q10, Jun 2015, Q4)

(a)	Area is $\int_0^a \frac{x^2(a-x)}{a^2}\,dx$	M1	
	$= \left[\frac{x^3}{3a} - \frac{x^4}{4a^2}\right]_0^a \quad (= \frac{a^2}{12})$	A1	
	$\int xy\,dx$	M1	
	$= \int_0^a \frac{x^3(a-x)}{a^2}\,dx = \left[\frac{x^4}{4a} - \frac{x^5}{5a^2}\right]_0^a \quad (= \frac{a^3}{20})$	A1	
	$\bar{x} = \frac{\frac{1}{20}a^3}{\frac{1}{12}a^2} = \frac{3a}{5}$	A1	
	$\int \frac{1}{2}y^2\,dx = \int_0^a \frac{x^4(a-x)^2}{2a^4}\,dx$	M1	For $\int \dots y^2\,dx$
	$= \left[\frac{x^5}{10a^2} - \frac{x^6}{6a^3} + \frac{x^7}{14a^4}\right]_0^a \quad (= \frac{a^3}{210})$	A2	Give A1 if just one error (e.g. omission of factor ½)
	$\bar{y} = \frac{\frac{1}{210}a^3}{\frac{1}{12}a^2} = \frac{2a}{35} \quad (\approx 0.0571a)$	A1	
		[9]	

(b)	**(i)**	Volume is $\int_0^3 \pi(x^2+16)\,dx$	M1	π may be omitted throughout	*Condone consistent use of $2\pi y^2$ etc*
		$= \pi\left[\frac{x^3}{3}+16x\right]_0^3 \quad (=57\pi)$	A1		
		$\int \pi x y^2\,dx$	M1		
		$= \int_0^3 \pi x(x^2+16)\,dx \quad = \pi\left[\frac{x^4}{4}+8x^2\right]_0^3 \quad (= \frac{369}{4}\pi)$	A1		
		$\bar{x} = \frac{\frac{369}{4}\pi}{57\pi} = \frac{123}{76} \quad (\approx 1.62)$	A1		
			[5]		
(b)	**(ii)**	Volume of A and B combined is $\pi \times 5^2 \times 3 = 75\pi$			
			M1	CM of composite body	
		$(18\pi)\bar{x}_B + (57\pi)\left(\frac{123}{76}\right) = (75\pi)(1.5)$	A2	Give A1 if just one error	FT values from (i)
	OR	$\int_0^3 \pi x\left(25-(x^2+16)\right)dx$		M1	
		$= \frac{81}{4}\pi$		A1	
		$(18\pi)\bar{x}_B = \frac{81}{4}\pi$		A1 FT	
		$\bar{x}_B = \frac{9}{8} \quad (=1.125)$	A1	CAO	
			[4]		