Q1, (Jun 2005, Q1)

A uniform solid cone has vertical height 20 cm and base radius $r$ cm. It is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted until the cone topples when the angle of inclination is $24^\circ$ (see diagram).

(i) Find $r$, correct to 1 decimal place.  \[4\]

A uniform solid cone of vertical height 20 cm and base radius 2.5 cm is placed on the plane which is inclined at an angle of $24^\circ$.

(ii) State, with justification, whether this cone will topple.  \[1\]

Q2, (Jan 2006, Q4)

A rectangular frame consists of four uniform metal rods. $AB$ and $CD$ are vertical and each is 40 cm long and has mass 0.2 kg. $AD$ and $BC$ are horizontal and each is 60 cm long. $AD$ has mass 0.7 kg and $BC$ has mass 0.5 kg. The frame is freely hinged at $E$ and $F$, where $E$ is 10 cm above $A$, and $F$ is 10 cm below $B$ (see diagram).

(i) Sketch a diagram showing the directions of the horizontal components of the forces acting on the frame at $E$ and $F$.  \[2\]

(ii) Calculate the magnitude of the horizontal component of the force acting on the frame at $E$.  \[3\]

(iii) Calculate the distance from $AD$ of the centre of mass of the frame.  \[3\]
A uniform solid hemisphere of weight 12 N and radius 6 cm is suspended by two vertical strings. One string is attached to the point O, the centre of the plane face, and the other string is attached to the point A on the rim of the plane face. The hemisphere hangs in equilibrium and OA makes an angle of 60° with the vertical (see diagram).

(i) Find the horizontal distance from the centre of mass of the hemisphere to the vertical through O. [2]

(ii) Calculate the tensions in the strings. [5]

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A uniform lamina \(ABCDE\) of weight 30 N consists of a rectangle and a right-angled triangle. The dimensions are as shown in the diagram.

(i) Taking x- and y-axes along \(AE\) and \(AB\) respectively, find the coordinates of the centre of mass of the lamina. [8]

The lamina is freely suspended from a hinge at \(B\).

(ii) Calculate the angle that \(AB\) makes with the vertical. [2]

The lamina is now held in a position such that \(BD\) is horizontal. This is achieved by means of a string attached to \(D\) and to a fixed point 15 cm directly above the hinge at \(B\).

(iii) Calculate the tension in the string. [3]
An object consists of a uniform solid hemisphere of weight 40 N and a uniform solid cylinder of weight 5 N. The cylinder has height $h$ m. The solids have the same base radius 0.8 m and are joined so that the hemisphere’s plane face coincides with one of the cylinder’s faces. The centre of the common face is the point $O$ (see Fig. 1). The centre of mass of the object lies inside the hemisphere and is at a distance of 0.2 m from $O$.

(i) Show that $h = 1.2$. 

One end of a light inextensible string is attached to a point on the circumference of the upper face of the cylinder. The string is horizontal and its other end is tied to a fixed point on a rough plane. The object rests in equilibrium on the plane with its axis of symmetry vertical. The plane makes an angle of 30° with the horizontal (see Fig. 2). The tension in the string is $T$ N and the frictional force acting on the object is $F$ N.

(ii) By taking moments about $O$, express $F$ in terms of $T$. 

(iii) Find another equation connecting $T$ and $F$. Hence calculate the tension and the frictional force.
A door is modelled as a lamina $ABCD$ consisting of a uniform rectangular section $ABDE$ of weight 60 N and a uniform semicircular section $BCD$ of weight 10 N and radius 40 cm. $AB$ is 200 cm and $AE$ is 80 cm. The door is freely hinged at $F$ and $G$, where $G$ is 30 cm below $B$ and $F$ is 30 cm above $A$ (see diagram).

(i) Find the magnitudes and directions of the horizontal components of the forces on the door at each of $F$ and $G$. \[4\]

(ii) Calculate the distance from $AE$ to the centre of mass of the door. \[6\]

Q7, (Jan 2010, Q3)

![Image](https://via.placeholder.com/150)

**Fig. 1**

A uniform conical shell has mass 0.2 kg, height 0.3 m and base diameter 0.8 m. A uniform hollow cylinder has mass 0.3 kg, length 0.7 m and diameter 0.8 m. The conical shell is attached to the cylinder, with the circumference of its base coinciding with one end of the cylinder (see Fig. 1).

(i) Show that the distance of the centre of mass of the combined object from the vertex of the conical shell is 0.47 m. \[4\]
The combined object is freely suspended from its vertex and is held with its axis horizontal. This is achieved by means of a wire attached to a point on the circumference of the base of the conical shell. The wire makes an angle of $80^\circ$ with the slant edge of the conical shell (see Fig. 2).

(ii) Calculate the tension in the wire. 

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A uniform lamina $ABCDE$ consists of a rectangle $ABDE$ and an isosceles triangle $BCD$ joined along their common edge. $AB = DE = 8\text{ cm}$, $AE = BD = 10\text{ cm}$ and $BC = CD = 13\text{ cm}$ (see diagram).

(i) Find the distance of the centre of mass of the lamina from $AE$. 

(ii) The lamina is freely suspended from $B$ and hangs in equilibrium. Calculate the angle that $BD$ makes with the vertical.
A uniform solid prism has cross-section $ABCDE$ in the shape of a rectangle measuring 20 cm by 4 cm joined to a semicircle of radius 8 cm as shown in the diagram. The centre of mass of the solid lies in this cross-section.

(i) Find the distance of the centre of mass of the solid from $AB$. \[5\]

The solid is placed with $AE$ on rough horizontal ground (so the object does not slide) and is in equilibrium with a horizontal force of magnitude 4 N applied along $CB$.

(ii) Find the greatest and least possible values for the weight of the solid. \[5\]

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10, (Jun 2016, Q3)

A uniform lamina $ABDC$ is bounded by two semicircular arcs $AB$ and $CD$, each with centre $O$ and of radii $3a$ and $a$ respectively, and two straight edges, $AC$ and $DB$, which lie on the line $AOB$ (see Fig. 1).

(i) Show that the distance of the centre of mass of the lamina from $O$ is $\frac{13a}{3\pi}$. \[5\]
The lamina has mass 3 kg and is freely pivoted to a fixed point at $A$. The lamina is held in equilibrium with $AB$ vertical by means of a light string attached to $B$. The string lies in the same plane as the lamina and is at an angle of $40^\circ$ below the horizontal (see Fig. 2).

(ii) Calculate the tension in the string.  

(iii) Find the direction of the force acting on the lamina at $A$.  

[3]  

[4]