

Discrete Random Variables (From OCR 4732)

Q1, (Jan 2006, Q3)

(i)	$\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}$ or $\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3}$ $\times 2$ or + $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3}$ $= \frac{1}{5}$ AG	M1 M1 M1 A1	4	or $\frac{1}{10}$ <u>from tree</u> add 2 equal products of 3 probs all correct Must see correct working NB incorrect methods eg $\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$
(ii)	Σxp $= 4$ $\Sigma x^2 p (= 17)$ $- \mu^2$ $= 1$	M1 A1 M1 M1 A1	5	≥ 3 terms added. Allow arith errors. ≥ 3 terms added. Allow arith errors Indep if +ve result $\Sigma(x-\mu)^2 p$ M2; 3 terms: M1 dep +ve result Σxp & $\Sigma x^2 p$, if \div eg 4: M0A0 (- μ^2 poss M1)
Total			9	

Q2, (Jun 2007, Q1)

$(0 \times 0.1) + 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.4$ $= 2(.0)$ $(0^2 \times 0.1) + 1 \times 0.2 + 2^2 \times 0.3 + 3^2 \times 0.4 (= 5)$ $- 2^2$ $= 1$	M1 A1 M1 M1 A1 5	5	≥ 2 non-zero terms correct eg $\div 4$: M0 ≥ 2 non-zero terms correct $\div 4$: M0 Indep, ft their μ . Dep +ve result $(-2)^2 \times 0.1 + (-1)^2 \times 0.2 + 0^2 \times 0.3 + 1^2 \times 0.4$: M2 ≥ 2 non-0 correct: M1 $\div 4$: M0
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Q3, (Jan 2008, Q6)

i	Σyp $= 2.3$ $\Sigma y^2 p (= 5.9)$ $- (\Sigma yp)^2$ $= 0.61$ oe	M1 A1 M1 M1 A1	5	≥ 2 terms added $\div 3$ or $\div 6$ etc: M0 ≥ 2 terms added $\div 3$ or $\div 6$ etc: M0 dep +ve result $(-1.3)^2 \times 0.2 + (-0.3)^2 \times 0.3 + 0.7^2 \times 0.5$: M2 one term correct: M1 Use of Z: MR, lose last A1 (2.55, 0.4475)
ii	$0.2 \times 0.25 + 0.3 \times 0.1$ or $0.05 + 0.03$ alone $= 0.08$ oe	M2 A1	3	M1 for one product eg correct $\times 2$: M1 or clearly ident (1,2), (2,1): M1
iii	$0.3 \times 0.1 + 0.3 \times 0.25 + 0.3 \times 0.65$ $+ 0.25 \times 0.2 + 0.25 \times 0.5$ alone or $0.03 + 0.075 + 0.195 + 0.05 + 0.125$ $= 0.475$ or $\frac{19}{40}$ oe	M2 A1	3	M1 : any 3, 4 of these prods alone or these 5 prods plus 1 extra or repeat or (ii) + prod or $0.3 + \text{prod}$ or $0.25 + \text{prod}$ or clearly identify (1,2) (3,2) (2,2) (2,1) (2,3) M2 for $0.3 + (0.2 + 0.5) \times 0.25$ or $0.25 + (0.1 + 0.65) \times 0.3$ or $0.3 + 0.25 - 0.3 \times 0.25$ or $1 - (0.2 + 0.5)(0.1 + 0.65)$ M1 for $(0.2 + 0.5)(0.1 + 0.65)$
Total			11	

Q4, (Jan 2009, Q1)

(i)	$0.2^2 + 0.7 \times 0.1 \times 2$ $= 0.18$ AG	M2	0.2^2 or 0.7×0.1 : M1
(ii)	$0.28 + 2 \times 0.18 + 3 \times 0.04 + 4 \times 0.01$ $= 0.8$ oe $0.28 + 2^2 \times 0.18 + 3^2 \times 0.04 + 4^2 \times 0.01$ - "0.8" ² $= 0.88$ oe	A1 3	no errors seen NB $2 \times 0.9 \times 0.1 = 0.18$ M0A0 ≥ 2 terms correct (excl 0×0.49) $\div 5$ (or 4 or 10 etc): M0 A1 ≥ 2 terms correct (excl $0^2 \times 0.49$) dep +ve result M1 dep +ve result M1 cao $\Sigma(x - \mu)^2$: 2 terms: M1; 5 terms M2 A1 5 $0.8^2 \times 0.49 + 0.2^2 \times 0.28 + 1.2^2 \times 0.18 + 2.2^2 \times 0.04 + 3.2^2 \times 0.01$ SC Use original table, 0.4:B1 0.44: B1

Q5, (Jun 2010, Q5)

i	$\frac{1}{2} \times \frac{1}{3}$ or $\frac{2}{4} \times \frac{1}{3}$ or $\frac{1}{4C_2}$ or $\frac{2}{12}$ $(= \frac{1}{6})$ AG $\frac{1}{4} \times \frac{2}{3}$ or $2 \times \frac{1}{4} \times \frac{1}{3}$ or $\frac{1}{2} \times \frac{1}{3}$ or $\frac{2}{4} \times \frac{1}{3}$ Add two of these or double one $(= \frac{1}{3})$ AG	B1	or 1 out of 6 or 2 out of 12 or $\frac{2!}{4!} \times 2$
		B1	or $\frac{2}{12}$ or $\frac{1}{6}$ or $\frac{1}{3!}$ or $\frac{1}{4C_2}$ or $\frac{2!}{4!} \times 2$
		B1 3	or $\frac{2}{4C_2}$ or $4 \times \frac{1}{4} \times \frac{1}{3}$ or $\frac{2}{4} \times \frac{2}{3}$ or $\frac{4}{12}$ or $\frac{2!}{4!} \times 4$ B1B1 or $\frac{2}{6}$ or $2 \times \frac{1}{6}$ or $\frac{2}{3!}$ or $\frac{2!}{3!}$ B1B1
ii	$X = 3, 4, 5, 6$ only, stated or used $P(X=5)$ wking as for $P(X=4)$ above or $1 - (\frac{1}{6} + \frac{1}{3} + \frac{1}{6})$ or $\frac{1}{3}$ $P(X=3)$ wking as for $P(X=6)$ above or $1 - (\frac{1}{3} + \frac{1}{3} + \frac{1}{6})$ or $\frac{1}{6}$ $\begin{matrix} 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{matrix}$ oe	B1	Allow repetitions Allow other values with zero probabilities.
		M1	
		M1	or M1 for total of their probs = 1, dep B1 or $P(X=3) = \frac{1}{6}, P(X=4) = \frac{1}{3}, P(X=5) = \frac{1}{3}, P(X=6) = \frac{1}{6}$
iii	Σxp $= 4 \frac{1}{2}$ $\Sigma x^2 p$ $(= 21 \frac{1}{6})$ - $4 \frac{1}{2}^2$ $= \frac{11}{12}$ or 0.917 (3 sf)	A1 4	Complete list of values linked to probs ≥ 2 terms correct ft ≥ 2 terms correct ft Independent except dependent on +ve result A1 5
Total		12	

Q6, (Jan 2011, Q7)

i	$(0 \times a) + 2 \times (1 - a)$ $= 2 - 2a$ or $2(1 - a)$ oe	M1 A1 2	or $2(1 - a)$ Not ISW	Condone $2 \times 1 - a$ NB $2 \times (1 - a) \div 2$: M0A0 Eg $E(X) = 2 - 2a$; $2 - 2a = 1$; $a = 0.5$: M1A0				
ii	$(0 \times a) + 2^2 \times (1 - a)$ $- "(2 - 2a)"^2$ $= 4 - 4a - 4 + 8a - 4a^2$ $= 4a - 4a^2$ $(= 4a(1 - a))$ AG	M1 M1 A1 3	or $4 - 4a$ oe $-(i)^2$ dep contains a ; ISW; Indep mk or $4(1 - a) - 4(1 - a)^2$ $4(1 - a)(1 - (1 - a))$	Condone $2^2 \times 1 - a$ $4 - 4a - 4 \pm 8a \pm 4a^2$ or $4 - 4a - 4 \pm 4a^2$ or equiv M1M1A0 $4 - 4a - 2(1 - a)^2$ M1M1A0 Must see this line, correctly obtained Careful: $4 - 4a - (2 - 2a)^2 = 4 - 4a - (4 - 4a^2) = -4a + 4a^2 = 4a(1 - a)$ M1M1A0 only				
<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>$-2 + 2a$</td> <td>$2a$</td> </tr> <tr> <td>a</td> <td>$1 - a$</td> </tr> </table>		$-2 + 2a$	$2a$	a	$1 - a$	M1	Correct table oe	
$-2 + 2a$	$2a$							
a	$1 - a$							
$\text{Var}(X) = a(-2+2a)^2 + 4a^2(1 - a)$ M1 $4a^3 - 8a^2 + 4a + 4a^2 - 4a^3$ $4a - 4a^2$ A1								
Total		5						

Q7, (Jun 2013, Q3)

(i)	$1 \times 0.4 + 3 \times 0.3 + 5 \times 0.2 + 7 \times 0.1$ $= 3$ $1^2 \times 0.4 + 3^2 \times 0.3 + 5^2 \times 0.2 + 7^2 \times 0.1$ $- "3"^{*2}$ $= 4$	M1 A1 M1 M1 A1 [5]	≥ 3 terms correct \div eg 4 M0 ≥ 3 terms correct \div eg 4 M0 Dep +ve result	Use of $\Sigma(x - \bar{x})^2 \times p$: $2^2 \times 0.4 + 0 + 2^2 \times 0.2 + 4^2 \times 0.1$ M2 or 2 correct non-zero terms M1
(ii)	$775, 757, 577$ $\frac{2}{3}$ or 0.667 (3 sf)	B1 B1 [2]	Must show all three	Allow repeats, eg list of 6 orders Alt method $X_1: 5$ or 7 , $X_2: 5$ or 7 ; $X_3: 5$ or 7 or X_1, X_2, X_3 can be 5 or 7 B1
(iii)	Binomial stated, or seen or implied with any n & p ${}^{11}C_4 \times 0.8^7 \times 0.2^4$ $= 0.111$ (3 sf)	B1 M1 A1 [3]	eg by $0.8^r \times 0.2^s$ ($r, s > 1$) not just by nC_r Correct method Correct answer, no working M1M1A1	NB 0.0388 scores B1M0A0 as it is ${}^{11}C_5 \times 0.8^6 \times 0.8^5$

Q8, (Jun 2014, Q2)

(a)		$(0^2 \times 0.3) + 2^2 \times 0.4 + 4^2 \times 0.3$ $- 2^2$ or -4 $= 2.4$	M1 M1 A1 [3]	last two terms correct. NOT eg $\div 6$ or $\div 3$ allow $-(\text{any number})^2$, dep +ve result	$2^2 \times 0.3 + (0) + 2^2 \times 0.3$ M2 1st or 3rd term correct M1 $\div 3$ M0M0A0
(b)	(i)	$2k + 3k + 4k + 5k = 1$ oe $(k = \frac{1}{14}$ AG)	B1 [1]	or $14k = 1$ oe " $= 1$ " is essential	NOT just $2 + 3 + 4 + 5 = 14$ so $k = \frac{1}{14}$ Allow verification, eg stating that $\frac{2}{14} + \frac{3}{14} + \frac{4}{14} + \frac{5}{14} = 1$
(b)	(ii)	$\frac{2}{14}, \frac{3}{14}, \frac{4}{14}, \frac{5}{14}$ or $\frac{2}{14}, \frac{6}{14}, \frac{12}{14}, \frac{20}{14}$ Σxp $= \frac{20}{7}$ or $2\frac{6}{7}$ or 2.86 (3 sf) oe, eg $\frac{40}{14}$	B1 M1 A1 [3]	≥ 3 correct ≥ 3 correct terms added SC $1 \times \frac{1}{14} + 2 \times \frac{2}{14} + 3 \times \frac{3}{14} + 4 \times \frac{4}{14} (=2.143)$ B0M1A0	$2k, 6k, 12k, 20k$ B1 $2k + 6k + 12k + 20k$ or $40k$ M1 $\div 4$ M0A0

Q9, (Jun 2015, Q9)

(i)		$a + b, a + 2b, a + 3b$ $a + b + a + 2b + a + 3b = 1$ oe $(3a + 6b = 1$ AG)	B1 B1dep [2]	All three seen Must see this line oe before final answer or "Probabilities add up to 1" oe stated	Must include " $= 1$ "
(ii)		$a + b + 2(a + 2b) + 3(a + 3b) = \frac{5}{3}$ $6a + 14b = \frac{5}{3}$ or $18a + 42b = 5$ eg $6 \times \frac{1-6b}{3} + 14b = \frac{5}{3}$ or $2b = -\frac{1}{3}$ or $6a + 14 \times \frac{1-3a}{6} = \frac{5}{3}$ or $3a = 2$ $a = \frac{2}{3}, b = -\frac{1}{6}$	M1 A1f A1 A1 [4]	fit their probs or any correct three term equn, fit their probs or any correct equn in a or b only. cao cao	

Q10, (Jan 2013, Q1)

(i)	$2k + 4k + 6k + 8k = 1$ $k = \frac{1}{20} \text{ AND } 6 \times \frac{1}{20} = \frac{3}{10} \quad \mathbf{AG}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>or $2 + 4 + 6 + 8 = 20$ M1</p> <p>Must see both for A1</p> <p>or $2k + 4k + 6k + 8k = 20k$ M1</p> $P(X = 6) = \frac{6k}{20k} = \frac{3}{10} \quad \text{A1}$	<p>Must see correct wk'g for $k = \frac{1}{20}$, otherwise M0A0</p> <p>NB $k \times 6 = \frac{3}{10} \Rightarrow k = \frac{1}{20}$ M0A0</p> <p>(even if tested by showing that $k = \frac{1}{20}$ gives $\Sigma p = 1$)</p> <p>Just showing $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = 1$ M0A0</p>
(ii)	$2 \times \frac{1}{10} + 4 \times \frac{2}{10} + 6 \times \frac{3}{10} + 8 \times \frac{4}{10} \text{ oe}$ $= 6$ $2^2 \times \frac{1}{10} + 4^2 \times \frac{2}{10} + 6^2 \times \frac{3}{10} + 8^2 \times \frac{4}{10} \text{ oe (= 10)}$ $- '6'^2$ $= 4$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>≥ 3 terms correct ft their values of p, dep $\Sigma p = 1$</p> <p>cao</p> <p>≥ 3 terms correct; ft their values of p; dep $\Sigma p = 1$</p> <p>ft their values of p; dep +ve result & $\Sigma p = 1$</p> <p>cao</p>	<p>Allow i.t.o. k for M1 $\div 4$ M0</p> <p>Allow i to k for M1M1 $\div 4$ M0</p> <p>NOT $-m^2 \div 4$</p> <p>$\sqrt{4} = 2$ lose final A1, not ISW, unless labelled sd</p>

Q11, (Jun 2016, Q1)

i	Σxp $= 2.7$ oe Σx^2p $- "2.7"^2$ $= 0.81$ oe (= 8.1)	M1 A1 M1 M1 A1 [5]	≥ 3 correct terms ≥ 3 correct terms dep +ve result	$\div 4$ or $\div 10$ etc M0A0 $(x - '2.7')$ ≥ 3 correct terms M1 $\Sigma(x - '2.7')^2p$ ≥ 3 correct terms M1 $\div 4$ or $\div 10$ etc M0M0 A0
ii	$0.3^2 \times 0.7 \times 3 + 0.3^3$ or $0.189 + 0.027$ oe $= \frac{27}{125}$ or 0.216	M2 A1 [3]	$0.3^2 \times 0.7 + 0.3^3$ M1 $0.3^2 \times 0.7 \times 3$ or 0.189 oe M1 0.3^2 or 0.09 : M0 unless clearly part of correct method $0.3^2 \times 0.1 \times 3 + 0.3^2 \times 0.4 \times 3 + 0.3^2 \times 0.2 \times 3 + 0.3^3$ M2 "x3" omitted 3 times or ≥ 2 terms correct M1	$1 - ({}^3C_1 \times 0.3 \times 0.7^2 + 0.7^3)$ M2 or $1 - (0.3 \times 0.7^2 + 0.7^3)$ M1 SC: M1 for $0.3^2 \times 0.6 \times 3 + 0.3^3$ or $1 - ({}^3C_1 \times 0.3 \times 0.6^2 + 0.6^3)$ \times^3C_1 or \times^3C_2 instead of $\times 3$ is OK throughout
		8		

Q12, (Jun 2008, Q4)

(i)	$\frac{1}{20} \times \frac{1}{10}$ or $\frac{1}{200}$ or 0.005 $\times 2$ $= \frac{1}{100}$ or 0.01	M1 M1dep A1 3	
(ii)	$E(X) = 0 + 50x^{1/10} + 500x^{1/20}$ or $0 + 0.5x^{1/10} + 5x^{1/20}$ $= 30p$ $= \pounds 0.30$ or $\frac{3}{10}$ Charge "30p" + 20p or $0.3 + 0.2$ $= 50p$ or 0.50 or 0.5	M1 A1 M1 A1 4	or eg 20 goes: $2 \times \pounds 0.50 + \pounds 5.00$ $= \pounds 6.00$ ("£6.00" + $20 \times \pounds 0.20$) $\div 20$ condone muddled units eg $0.3 + 20$ $x = 20, 70, 520$: M1A1 $20 \times \frac{17}{20} + 70 \times \frac{1}{10} + 520 \times \frac{1}{20}$: M1 $= 50$ A1 $x, (x - 50), (x - 500)$: M1A1 $x \times \frac{17}{20} + (x - 50) \times \frac{1}{10} + (x - 500) \times \frac{1}{20} = 20$: M1 $x = 50$: A1 Ignore "£" or "p"
Total		7	

Q13, (Jun 2012, Q6)

$$(1 - 0.1) \div 5 \quad (= 0.18)$$

$$3 \times 0.18 \text{ or } 2 \times 0.18 \text{ or } 7 \times 0.1 \text{ (or result of these)(poss } \times 100)$$

(3×0.18 only scores if using £3, not score of 3. Similarly for 2×0.18).

$$4 \times 3 \times 0.18 \text{ AND } 2 \times 0.18 + 7 \times 0.1 \text{ (poss } \times 100)$$

(or 2.16 AND 1.06 or 216 AND 106)

$$'2.16' - '1.06' \text{ or } '216' - '106'$$

must be attempt gain on 1,2,3,4 – loss on 5,6

$$E(\text{profit for 100 rolls}) = (\pounds)110$$

M1 can be implied, eg by 18

M1 $5 \times 0.18 \text{ or } 10 \times 0.1$ (or result of these)(poss $\times 100$)

M1 3 AND $5 \times 0.18 + 10 \times 0.1$ (poss $\times 100$)
(or 3 AND 1.9 or 300 AND 190)

M1 3 – '1.9' or 300 – '190'
^{dep}
any M1 must be attempt receipt – payout on 5,6

A1 $E(\text{profit for 100 rolls}) = (\pounds)110$

NB $300 - (0.1 \times 300 + 0.18 \times 300) = 300 - 84 = 216$
[5] M1M1M0M0A0

or, using exp no. of 5's & 6's
 $18 \times 5 \text{ or } 10 \times 10$

300 AND $18 \times 5 + 10 \times 10$
(NB $300 + 100 \times 0.18 + 100 \times 0.1$ is insuff)

Eg:
 $300 - 100 \times (5 \times 0.18 + 6 \times 0.1) = 150$
M1M1M0M1A0

Mark one method only
Must be matched pair
eg $300 - 106$ or $216 - 190$:
M1M1M0M0A0