Vectors Exam Questions (From OCR 4724)

Q1, (Jun 2005, Q3)

The line L_1 passes through the points $(2, -3, 1)$ and $(-1, -2, -4)$. The line L_2 passes through the point $(3, 2, -9)$ and is parallel to the vector $4\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

(i) Find an equation for L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [2]

(ii) Prove that L_1 and L_2 are skew. [5]

Q2, (Jun 2008, Q6)

Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}.$$

(i) Show that the lines intersect. [4]

(ii) Find the angle between the lines. [4]

Q3, (Jan 2010, Q2)

Points A, B and C have position vectors $-5\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$, $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + p\mathbf{k}$ respectively, where p is a constant.

(i) Given that angle $ABC = 90°$, find the value of p. **[4]**

(ii) Given instead that ABC is a straight line, find the value of p. **[2]**

Q4, (Jan 2010, Q9)

The equation of a straight line l is $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. O is the origin.

(i) The point P on l is given by $t = 1$. Calculate the acute angle between OP and l. **[4]**

(ii) Find the position vector of the point Q on l such that OQ is perpendicular to l. **[4]**

(iii) Find the length of OQ. **[2]**

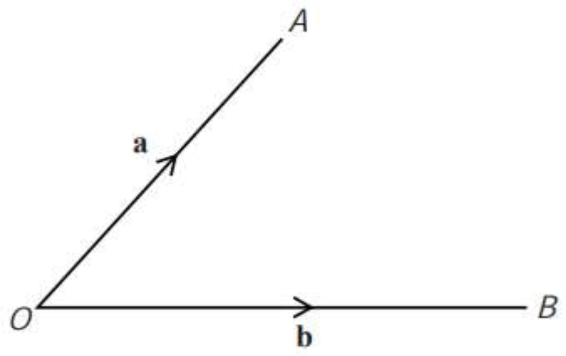
Q5, (Jun 2013, Q3)

Determine whether the lines whose equations are

$$\mathbf{r} = (1 + 2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3 + 5\lambda)\mathbf{k} \quad \text{and} \quad \mathbf{r} = (\mu - 1)\mathbf{i} + (5 - \mu)\mathbf{j} + (2 - 5\mu)\mathbf{k}$$

are parallel, intersect or are skew. **[6]**

Q6, (Jun 2012, Q5)



In the diagram the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ with respect to the origin O. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $\mathbf{a}.\mathbf{b} = 6$, find

(i) the angle AOB, **[2]**

(ii) $|\mathbf{a} - \mathbf{b}|$. **[3]**

Q7, (Jan 2013, Q8)

The points $A\ (3, 2, 1)$, $B\ (5, 4, -3)$, $C\ (3, 17, -4)$ and $D\ (1, 6, 3)$ form a quadrilateral $ABCD$.

(i) Show that $AB = AD$. **[2]**

(ii) Find a vector equation of the line through A and the mid-point of BD. **[3]**

(iii) Show that C lies on the line found in part **(ii)**. **[1]**

(iv) What type of quadrilateral is $ABCD$? **[1]**

Q8, (OCR Y531, Sample Question Paper, Q9)

(i) Find the value of k such that $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ k \end{pmatrix}$ are perpendicular. **[2]**

Two lines have equations $l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(ii) Find the point of intersection of l_1 and l_2. **[4]**

(iii) The vector $\begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$ is perpendicular to the lines l_1 and l_2.

Find the values of a and b. **[5]**

Q9, (OCR 4727, Jan 2008, Q3)

For this question you may use the following property of the vector product where $\boldsymbol{a}$ and $\boldsymbol{b}$ refer to two general vectors, θ refers to the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ and $\hat{\boldsymbol{n}}$ is a unit vector perpendicular to $\boldsymbol{a}$ and $\boldsymbol{b}$:

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \sin\theta\,\hat{\mathbf{n}}$

Two fixed points, A and B, have position vectors $\mathbf{a}$ and $\mathbf{b}$ relative to the origin O, and a variable point P has position vector $\mathbf{r}$.

(i) Give a geometrical description of the locus of P when $\mathbf{r}$ satisfies the equation $\mathbf{r} = \lambda\mathbf{a}$, where $0 \leqslant \lambda \leqslant 1$. [2]

(ii) Given that P is a point on the line AB, use a property of the vector product to explain why $(\mathbf{r} - \mathbf{a}) \times (\mathbf{r} - \mathbf{b}) = \mathbf{0}$. [2]

(iii) Give a geometrical description of the locus of P when $\mathbf{r}$ satisfies the equation $\mathbf{r} \times (\mathbf{a} - \mathbf{b}) = \mathbf{0}$. [3]

Q10, (OCR 4727, Jun 2008. Q5)

Two lines have equations

$$\frac{x-k}{2} = \frac{y+1}{-5} = \frac{z-1}{-3} \quad \text{and} \quad \frac{x-k}{1} = \frac{y+4}{-4} = \frac{z}{-2},$$

where k is a constant.

(i) Show that, for all values of k, the lines intersect, and find their point of intersection in terms of k. [6]

Q11, (OCR 4727, Jan 2010, Q1)

Determine whether the lines

$$\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z+4}{2} \quad \text{and} \quad \frac{x+3}{2} = \frac{y-1}{3} = \frac{z-5}{4}$$

intersect or are skew. **[5]**
