

Vectors Exam Questions (From OCR 4724)

Q1, (Jun 2005, Q3)

(i)	M1	For (either point) + t(diff betw vectors)
$\mathbf{r} = (2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ or $-\mathbf{i}-2\mathbf{j}-4\mathbf{k}) + t(3\mathbf{i}-\mathbf{j}+5\mathbf{k})$	A1 2	Completely correct including $\mathbf{r} =$. AEF
(ii) $L(2)$ $(\mathbf{r}) = 3\mathbf{i}+2\mathbf{j}-9\mathbf{k}+s(4\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$	M1	For point + (s or t) direction vector
$L(1)$ & $L(2)$ must be of form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$		
$2+3t=3+4s, -3-t=2-4s, 1+5t= -9+5s$	M1	For 2/3 eqns with 2 different parameters
or suitable equivalences		
$(t,s) = (+/-3,2)$ or $(-/+1,1)$ or $(-/+9,-7)$	M1	For solving any relevant pair of eqns
or $(+/-4,2)$ or $(0,1)$ or $(-/+8,-7)$	A1	For both parameters correct
Basic check other eqn & interp \checkmark	B1 5	

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Q2, (Jun 2008, Q6)

(i) Produce at least 2 of the 3 relevant equations in t and s	M1	$1 + 2t = 12 + s, 3t = -4s, -5 + 4t = 5 - 2s$
Solve for t and s	M1	
$(t, s) = (4, -3)$ AEF	*A1	
Subst $(4, -3)$ into suitable equation(s) & show consistency	dep*A1	Either into "3 rd " eqn or into all 3 coordinates. N.B. Intersection coords not asked for

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(ii) Method for finding magnitude of any vector	*M1	Expect $\sqrt{29}$ and $\sqrt{21}$
Method for finding scalar product of any 2 vectors	*M1	Expect -18
Using $\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{ \mathbf{a} \mathbf{b} }$ AEF for the correct 2 vectors	dep*M1	Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$
137 (136.8359) or 43.2(43.164...)	A1	2.39 (2.388236...) or 0.753(0.75335...) rads

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Q3, (Jan 2010, Q2)

(i) Find at least 2 of $(\overrightarrow{AB}$ or $\overrightarrow{BA}), (\overrightarrow{BC}$ or $\overrightarrow{CB}), (\overrightarrow{AC}$ or $\overrightarrow{CA})$	M1	irrespective of label; any notation
Use correct method to find scal prod of any 2 vectors	M1	or use corr meth for modulus
Use $\overrightarrow{AB}\cdot\overrightarrow{BC} = 0$ or $\frac{\overrightarrow{AB}\cdot\overrightarrow{BC}}{ \overrightarrow{AB} \overrightarrow{BC} } = 0$	M1	or use $ \overrightarrow{AB} ^2 + \overrightarrow{BC} ^2 = \overrightarrow{AC} ^2$
Obtain $p=1$ (dep 3 @ M1)	A1 4	

(ii) Use equal ratios of appropriate vectors	M1	or scalar product method
Obtain $p=-8$	A1 2	

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Q4, (Jan 2010, Q9)

(i) P is $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ B1

direction vector of ℓ is $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and of \overrightarrow{OP} is their P $\sqrt{B1}$

Use $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$ for $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and their OP M1

$\theta = 35.3$ or better (0.615... rad) A1 4

(ii) Use $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3+t \\ 1-t \\ 1+2t \end{pmatrix} = 0$ M1

$1(3+t) - 1(1-t) + 2(1+2t) = 0$ A1

$t = -\frac{2}{3}$ A1

Subst. into $\begin{pmatrix} 3+t \\ 1-t \\ 1+2t \end{pmatrix}$ to produce $\begin{pmatrix} 7/3 \\ 5/3 \\ -1/3 \end{pmatrix}$ ISW A1 4

(iii) Use $\sqrt{x^2 + y^2 + z^2}$ where $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is part (ii) answer M1

Obtain $\sqrt{\frac{75}{9}}$ AEF, 2.89 or better (2.8867513....) A1 2

Q5, (Jun 2013, Q3)

Set up the 3 relevant equations
 $1 + 2\lambda = \mu - 1$ $-\lambda = 5 - \mu$ $3 + 5\lambda = 2 - 5\mu$

Attempt to find λ or μ from 2 of the equations & then find μ or λ from any of the 3 equations.

$(\lambda, \mu) = (3, 8)$ or $(-2\frac{3}{5}, 2\frac{2}{5})$ or $(-\frac{11}{15}, \frac{8}{15})$
 or $(3, -3\frac{1}{5})$ or $(-\frac{11}{15}, 4\frac{4}{15})$ or $(-2\frac{3}{5}, -3\frac{1}{5})$
 or $(\frac{1}{5}, 2\frac{2}{5})$ or $(-8\frac{1}{5}, 8)$ or $(-4\frac{7}{15}, \frac{8}{15})$

Demonstrate inconsistency i.e. substitute the correct values into a correct equation (but not the immediate last one used)

State “skew”

(a) Identify direction vectors; (b) state “not identical/same/equal/equiv/multiples” or eval $\cos(\text{angle})$ & state $\neq 1$ (or -1); (c) state “not parallel”

M1	‘M’ mark so intention must be clear; minor error(s) only accepted
M1	Or find λ , say, from (i)(ii) & then from (ii)(iii) [values shown at next stage] – inconsistency dep*A1 also awarded here
A1	Accept equivalent proper/improper fractional values or decimal equivalent values
M1	e.g. after (3,8), subst in iii & write $3 + 5 \times 3 \neq 2 - 5 \times 8$ or $3 + 5 \times 3 = 2 - 5 \times 8 \therefore$ do not intersect
A1	Dep on 3 @ M1 + A1
B1	dvs <u>must be identified</u> : $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$
	Accept any vector notation.
[6]	

Q6, (Jun 2012, Q5)

(i)	Use $\cos \theta = \frac{a \cdot b}{ a b }$	M1
	Obtain $\left(\cos \theta = \frac{6}{12} \right) \theta = 60$ or $\frac{1}{3}\pi$ or 1.05 or better	A1
		[2]
(ii)	Indicate $\mathbf{a} - \mathbf{b}$ is vector joining ends of \mathbf{a} and \mathbf{b} or equiv $ \mathbf{a} - \mathbf{b} = \mathbf{a} - \mathbf{b} $, or anything similar, \rightarrow M0	M1
	Use cosine rule correctly on 3, 4 and included (i) angle	M1
	Obtain $\sqrt{13}$ or 3.61 or better (No ft from wrong θ)	A1
		[3]

Q7, (Jan 2013, Q8)

(i)	$AB = \sqrt{(+/-2)^2 + (+/-2^2 + (+/-4)^2)}$	B1	oe
	$AD = \sqrt{(+/-2)^2 + (+/-4)^2 + (+/-2)^2}$	B1	oe
		[2]	
(ii)	midpoint is (3, 5, 0)	B1	Accept any reasonable vector notation.
	Clear method for finding direction vector	M1	Expect $3\mathbf{j} - \mathbf{k}$ or $-3\mathbf{j} + \mathbf{k}$
	$\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(3\mathbf{j} - \mathbf{k})$ oe or e.g. $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} + \mu(-3\mathbf{j} + \mathbf{k})$ cao	A1	" $\mathbf{r} =$ " is essential. No f.t. for wrong mid-point.
		[3]	
(iii)	substitution of $\lambda = +/-5$ or $\mu = +/-4$	M1	Based on correct answer to (ii)
		[1]	
(iv)	Kite	B1	
		[1]	

Q8, (OCR Y531, Sample Question Paper, Q9)

(i)	$-1 \times -2 + 2 \times 3 + 1 \times k = 0$ $\Rightarrow k = -4$	M1	1.1	Attempt the scalar product and set equal to zero so i
		A1	1.1	
		[2]		
(ii)	Equate x and y coordinates: $3 + \lambda = 6 + 2\mu \Rightarrow \lambda - 2\mu = 3$ $2 - \lambda = 5 + \mu \Rightarrow \lambda + \mu = -3$ $\Rightarrow \mu = -2, \lambda = -1$ Consistent with z coordinates since $7 + 3 \times (-1) = 4$ and $2 - (-2) = 4$ So the point of intersection is (2, 3, 4)	M1	2.1	Use coordinates to find μ and λ .
		A1	1.1	
		E1	1.1	Check consistency with third coordinate
		A1	1.1	
		[4]		
(iii)	The vector product find a mutual perpendicular $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 7 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = \lambda \begin{pmatrix} -2 \\ 7 \\ 3 \end{pmatrix}$ $\lambda = -\frac{1}{2}$ $a = 1, b = -1.5$ $a = -3.5$	M1	3.1a	Attempt the vector product, by any valid method
		A1	1.1	BC
		M1	3.1a	
		M1	1.1	
		A1	1.1	
	[5]			

(i) line segment OA	B1 B1 2	For stating line through O OR A For correct description AEF
(ii) $(\mathbf{r} - \mathbf{a}) \times (\mathbf{r} - \mathbf{b}) = \vec{AP} \times \vec{BP}$ $= AP BP \sin \pi \cdot \hat{\mathbf{n}} = \mathbf{0}$	B1 B1 2	For identifying $\mathbf{r} - \mathbf{a}$ with \vec{AP} and $\mathbf{r} - \mathbf{b}$ with \vec{BP} Allow direction errors For using \times of 2 parallel vectors = $\mathbf{0}$ OR $\sin \pi = 0$ or $\sin 0 = 0$ in an appropriate vector expression
(iii) line through O parallel to AB	B1 B1 B1 3 <div style="border: 1px solid black; width: 15px; height: 15px; display: inline-block; text-align: center; line-height: 15px;">7</div>	For stating line For stating through O For stating correct direction SR For \vec{AB} or \vec{BA} allow B1 B0 B1

Q10, (OCR 4727, Jun 2008, Q5i)

METHOD 1

Lines meet where

$$(x =) k + 2\lambda = k + \mu$$

$$(y =) -1 - 5\lambda = -4 - 4\mu$$

$$(z =) 1 - 3\lambda = -2\mu$$

$$\Rightarrow \lambda = -1, \mu = -2$$

$$\Rightarrow (k - 2, 4, 4)$$

M1	For using parametric form to find where lines meet
A1	For at least 2 correct equations
M1	For attempting to solve any 2 equations
A1	For correct values of λ and μ
B1	For attempting a check in 3rd equation OR verifying point of intersection is on both lines
A1 6	For correct point of intersection (allow vector) SR For finding λ OR μ and point of intersection, but check, award up to M1 A1 M1 A0 B0 A1

METHOD 2

$$d = \frac{[0, 3, 1] \cdot [2, -5, -3] \times [1, -4, -2]}{|\mathbf{b} \times \mathbf{c}|}$$

$$d = c[0, 3, 1] \cdot [-2, 1, -3] = 0$$

\Rightarrow lines intersect

Lines meet where

$$(x =) (k+) 2\lambda = (k+) \mu$$

$$(y =) -1 - 5\lambda = -4 - 4\mu$$

$$(z =) 1 - 3\lambda = -2\mu$$

$$\Rightarrow \lambda = -1, \mu = -2$$

$$\Rightarrow (k - 2, 4, 4)$$

B1	For using $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ with appropriate vectors (division by $ \mathbf{b} \times \mathbf{c} $ is not essential) and showing $d = 0$ correctly
M1	For using parametric form to find where lines meet
A1	For at least 2 correct equations
M1	For attempting to solve any 2 equations
A1	For correct value of λ OR μ
A1	For correct point of intersection (allow vector)

METHOD 3

$$\text{e.g. } x - k = \frac{2(y+1)}{-5} = \frac{y+4}{-4}$$

$$\Rightarrow y = 4$$

$$\frac{z-1}{-3} = \frac{y+1}{-5}$$

$$x = k - 2 \text{ OR } z = 4$$

$$x - k = \frac{z}{-2} \text{ checks with } x = k - 2, z = 4$$

$$\Rightarrow (k - 2, 4, 4)$$

M1	For solving one pair of simultaneous equations
A1	For correct value of x, y or z
M1	For solving for the third variable
A1	For correct values of 2 of x, y and z
B1	For attempting a check in 3rd equation
A1	For correct point of intersection (allow vector)

Q11, (OCR 4727, Jan 2010, Q1)

METHOD 1

line segment between l_1 and $l_2 = \pm[4, -3, -9]$

$$\mathbf{n} = [1, -1, 2] \times [2, 3, 4] = (\pm)[-2, 0, 1]$$

$$\text{distance} = \frac{|[4, -3, -9] \cdot [-2, 0, 1]|}{\left(\sqrt{2^2 + 0^2 + 1^2}\right)} = \frac{17}{(\sqrt{5})}$$

$\neq 0$, so skew

- B1 For correct vector
- M1* For finding vector product of direction vectors
- A1
- M1 For using numerator of distance formula
- (*dep)
- A1 **5** For correct scalar product and correct conclusion

METHOD 2 lines would intersect where

$$\left. \begin{array}{l} 1 + s = -3 + 2t \\ -2 - s = 1 + 3t \\ -4 + 2s = 5 + 4t \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} s - 2t = -4 \\ s + 3t = -3 \\ 2s - 4t = 9 \end{array} \right.$$

\Rightarrow contradiction, so skew

- B1 For correct parametric form for either line
- M1* For 3 equations using 2 different parameters
- A1
- M1 For attempting to solve to show (in)consistency
- (*dep)
- A1 For correct conclusion