Q1, (Jun 2006, Q3)
One root of the quadratic equation $x^2 + px + q = 0$, where $p$ and $q$ are real, is the complex number $2 - 3i$.

(i) Write down the other root. [1]

(ii) Find the values of $p$ and $q$. [4]

Q2, (Jun 2005, Q8)
(a) The quadratic equation $x^2 - 2x + 4 = 0$ has roots $\alpha$ and $\beta$.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = -4$. [2]

(iii) Hence find a quadratic equation which has roots $\alpha^2$ and $\beta^2$. [3]

(b) The cubic equation $x^3 - 12x^2 + ax - 48 = 0$ has roots $p$, $2p$ and $3p$.

(i) Find the value of $p$. [2]

(ii) Hence find the value of $a$. [2]

Q3, (Jun 2007, Q6)
The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

(i) (a) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma + \gamma\alpha$. [2]

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(ii) (a) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in $u$ with integer coefficients. [2]

(b) Use your answer to part (ii) (a) to find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [2]

Q4, (Jun 2009, Q4)
The roots of the quadratic equation $x^2 + x - 8 = 0$ are $p$ and $q$. Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. [4]

Q5, (Jun 2009, Q5)
The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

(i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in $u$ with integer coefficients. [3]

(ii) Hence find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. [2]

Q6, (Jun 2010, Q7)
The quadratic equation $x^2 + 2kx + k = 0$, where $k$ is a non-zero constant, has roots $\alpha$ and $\beta$. Find a quadratic equation with roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha + \beta}{\beta}$. [7]
Q7, (Jan 2011, Q9)

The quadratic equation \(2x^2 - x + 3 = 0\) has roots \(\alpha\) and \(\beta\), and the quadratic equation \(x^2 - px + q = 0\) has roots \(\alpha + \frac{1}{\alpha}\) and \(\beta + \frac{1}{\beta}\).

(i) Show that \(p = \frac{5}{6}\). [4]

(ii) Find the value of \(q\). [5]

Q8, (Jan 2013, Q9)

(i) Show that \((\alpha\beta + \beta\gamma + \gamma\alpha)^2 = \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)\). [3]

(ii) It is given that \(\alpha, \beta\) and \(\gamma\) are the roots of the cubic equation \(x^3 + px^2 - 4x + 3 = 0\), where \(p\) is a constant. Find the value of \(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\) in terms of \(p\). [5]

Q9, (Jun 2014, Q5)

The cubic equation \(2x^3 + 3x + 3 = 0\) has roots \(\alpha, \beta\) and \(\gamma\).

(i) Use the substitution \(x = u + 2\) to find a cubic equation in \(u\). [3]

(ii) Hence find the value of \(\frac{1}{\alpha - 2} + \frac{1}{\beta - 2} + \frac{1}{\gamma - 2}\). [4]

Q10, (Jun 2014, Q9)

The roots of the equation \(x^3 - kx^2 - 2 = 0\) are \(\alpha, \beta\) and \(\gamma\), where \(\alpha\) is real and \(\beta\) and \(\gamma\) are complex.

(i) Show that \(k = \alpha - \frac{2}{\alpha^2}\). [2]

(ii) Given that \(\beta = u + iv\), where \(u\) and \(v\) are real, find \(u\) in terms of \(\alpha\). [4]

(iii) Find \(v^2\) in terms of \(\alpha\). [4]

Q11, (Jun 2015, Q10)

The cubic equation \(x^3 + 4x + 3 = 0\) has roots \(\alpha, \beta\) and \(\gamma\).

(i) Use the substitution \(x = \sqrt{u}\) to obtain a cubic equation in \(u\). [3]

(ii) Find the value of \(\alpha^4 + \beta^4 + \gamma^4 + \alpha\beta\gamma\). [7]

Q12, (Jun 2016, Q10)

(i) Use an algebraic method to find the square roots of the complex number \(9 + 40i\). [6]

(ii) Show that \(9 + 40i\) is a root of the quadratic equation \(z^2 - 18z + 1681 = 0\). [1]

(iii) By using the substitution \(z = \frac{1}{u^2}\), find the roots of the equation \(1681u^4 - 18u^2 + 1 = 0\). Give your answers in the form \(x + iy\), where \(x\) and \(y\) are real. [4]
Q13, (OCR 2605, Jan 2002, Q1)

The equation $x^4 - 6x^3 - 73x^2 + kx + m = 0$ has two positive roots $\alpha, \beta$ and two negative roots $\gamma, \delta$. It is given that $\alpha \beta = \gamma \delta = 4$.

(i) Find the values of the constants $k$ and $m$. [5]

(ii) Show that $(\alpha + \beta)(\gamma + \delta) = -81$. [4]

(iii) Find the quadratic equation which has roots $\alpha + \beta$ and $\gamma + \delta$. [2]

(iv) Find $\alpha + \beta$ and $\gamma + \delta$. [3]

(v) Show that $\alpha^2 - 3(1 + \sqrt{10})\alpha + 4 = 0$, and find similar quadratic equations satisfied by $\beta, \gamma$ and $\delta$. [6]

Q14, (OCR 2605, Jan 2004, Q1b)

The equation $x^4 + 4x^3 + 3x - 5 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

Using a substitution, or otherwise, find a quartic equation with integer coefficients which has roots $\alpha^2 - 1, \beta^2 - 1, \gamma^2 - 1, \delta^2 - 1$. [8]

Q15, (OCR 2605, Jan 2005, Q1a)

The equation $8x^4 + 16x^3 + 1 = 0$ has roots $\alpha, \beta, \gamma$ and $\delta$.

Use a suitable substitution to find a quartic equation with integer coefficients which has roots $8\alpha^3, 8\beta^3, 8\gamma^3$ and $8\delta^3$. [5]