Proof By Induction (Matrices) Exam Questions (From OCR 4725 unless otherwise stated)

Q1, (Jun 2005, Q9iv)

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}.$$

Prove by induction that $\mathbf{M}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}$, for all positive integers n. [6]

Q2, (Jun 2006, Q7)

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{A}^2$ and $\mathbf{A}^3$. [3]

(ii) Hence suggest a suitable form for the matrix $\mathbf{A}^n$. [1]

(iii) Use induction to prove that your answer to part **(ii)** is correct. [4]

Q3, (Jun 2008, Q4)

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geqslant 1$,

$$\mathbf{A}^n = \begin{pmatrix} 3^n & \frac{1}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}.$$ [6]

Q4, (Jan 2010, Q10)

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{M}^2$ and $\mathbf{M}^3$. **[3]**

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$. **[1]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[4]**

(iv) Describe fully the single geometrical transformation represented by $\mathbf{M}^{10}$. **[3]**

Q5, (Jan 2012, Q7)

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.

(i) Show that $\mathbf{M}^4 = \begin{pmatrix} 81 & 0 \\ 80 & 1 \end{pmatrix}$. **[3]**

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$, where n is a positive integer. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[4]**

Q6, (Jun 2013, Q4)

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geqslant 1$,

$$\mathbf{M}^n = \begin{pmatrix} 2^n & 2^{n+1} - 2 \\ 0 & 1 \end{pmatrix}.$$

[6]

Q7, (OCR 2604, Jun 2003, Q4b)

A sequence of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots$ is defined by $\mathbf{v}_1 = \begin{pmatrix} 11 \\ 13 \\ -4 \end{pmatrix}$ and $\mathbf{v}_{n+1} = \mathbf{M}\mathbf{v}_n$ for $n \geqslant 1$, where

$$\mathbf{M} = \begin{pmatrix} 1 & 3 & 3 \\ 0 & 4 & 3 \\ -3 & 0 & -2 \end{pmatrix}.$$

Show by induction that $\mathbf{v}_n = \begin{pmatrix} 9n^2 + 2 \\ 9n^2 + 4 \\ -(3n-1)^2 \end{pmatrix}$. **[8]**
