Proof By Induction (Matrices) Exam Questions (From OCR 4725)

Q1, (Jun 2005, Q9iv)

	B1		Explicit check for n = 1or n = 2
$\mathbf{M}^k = \begin{pmatrix} 2^k & 3(2^k - 1) \\ 0 & 1 \end{pmatrix}$.	M1		Induction hypothesis that result is true for $\mathbf{M}^k$
	M1		Attempt to multiply $\mathbf{MM}^k$ or vice versa
$\begin{pmatrix} 2^{k+1} & 3(2^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$.	A1 A1		Element $3(2^{k+1} - 1)$ derived correctly All other elements correct
	A1	6	Explicit statement of induction conclusion

Q2, (Jun 2006, Q7)

(i)	M1		Attempt at matrix multiplication
$\mathbf{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{A}^3 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$	A1 A1	 3	Correct $\mathbf{A}^2$ Correct $\mathbf{A}^3$
(ii) $\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$	B1	1	Sensible conjecture made
(iii)	B1 M1 A1 A1	 4 **8**	State that conjecture is true for n = 1 or 2 Attempt to multiply $\mathbf{A}^n$ and $\mathbf{A}$ or vice versa Obtain correct matrix Statement of induction conclusion

Q3, (Jun 2008, Q4)

B1	Establish result is true, for n = 1 (or 2 or 3)
M1	Attempt to multiply **A** and $\mathbf{A}^n$, or vice versa
M1	Correct process for matrix multiplication
A1	Obtain 3^{n+1}, 0 and 1
A1	Obtain $\frac{1}{2}(3^{n+1} - 1)$
A1	Statement of Induction conclusion, only if 5 marks earned, but may be in body of working
6	

Q4, (Jan 2010, Q10)

(i)

$\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ $\mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$

B1		Correct $\mathbf{M}^2$ seen
M1		Convincing attempt at matrix multiplication for $\mathbf{M}^3$
A1	**3**	Obtain correct answer

(ii) $\mathbf{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$ B1ft **1** State correct form, consistent with **(i)**

(iii)	M1		Correct attempt to multiply **M** & $\mathbf{M}^k$ or v.v.
	A1		Obtain element 2(k + 1)
	A1		Clear statement of induction step, from correct working
	B1	**4**	Clear statement of induction conclusion, following their working
(iv)	B1		Shear
	DB1		x-axis invariant
	DB1	**3**	e.g. (1, 1) → (21, 1) or equivalent using scale factor or angles
	11		

Q5, (Jan 2012, Q7)

(i)		M1 A1 A1 **[3]**	Attempt at matrix multiplication Obtain $\mathbf{M}^2$ correctly Obtain **given answer** correctly
(ii)	$\begin{pmatrix} 3^n & 0 \\ 3^n - 1 & 1 \end{pmatrix}$	B1 B1 **[2]**	3 elements correct 4[th] element correct
(iii)	$\begin{pmatrix} 3^{k+1} & 0 \\ 3^{k+1} - 1 & 1 \end{pmatrix}$	B1 M1 A1 B1 **[4]**	Show that their result is true for $n = 1$ or 2 Attempt to find $\mathbf{M}^k\mathbf{M}$ or vice versa Obtain correct answer Complete statement of induction conclusion

Q6, (Jun 2013, Q4)

	B1	Establish result true for $n = 1$ or $n = 2$
	M1	Multiply **M** and $\mathbf{M}^k$, either order
$2(2^{k+1} - 2) + 2$ or $2^{k+1} + 2^{k+1} - 2$	A1	Obtain correct element
	A1	Obtain other 3 correct elements
	A1	Obtain $2^{k+2} - 2$ convincingly
	B1	Specific statement of induction conclusion, provided 5/5 earned so far and verified for $n = 1$
	[6]	

Q7, (OCR 2604, Jun 2003, Q4b)

Solution	Mark	Guidance
When $n=1$, $\begin{pmatrix} 9n^2+2 \\ 9n^2+4 \\ -(3n-1)^2 \end{pmatrix} = \begin{pmatrix} 11 \\ 13 \\ -4 \end{pmatrix}$	B1	
so it is true for $n=1$ Assuming it is true for $n=k$,		
$\mathbf{v}_{k+1} = \begin{pmatrix} 1 & 3 & 3 \\ 0 & 4 & 3 \\ -3 & 0 & -2 \end{pmatrix} \begin{pmatrix} 9k^2+2 \\ 9k^2+4 \\ -(3k-1)^2 \end{pmatrix}$	M1	
$= \begin{pmatrix} 9k^2+2+3(9k^2+4)-3(3k-1)^2 \\ 4(9k^2+4)-3(3k-1)^2 \\ -3(9k^2+2)+2(3k-1)^2 \end{pmatrix}$	A2	Give A1 for one component correct
$= \begin{pmatrix} 9k^2+18k+11 \\ 9k^2+18k+13 \\ -9k^2-12k-4 \end{pmatrix}$	M1	Obtaining simple quadratic (one component sufficient) *Dependent on previous M1*
$= \begin{pmatrix} 9(k+1)^2+2 \\ 9(k+1)^2+4 \\ -(3k+2)^2 \end{pmatrix}$	A2	Correctly obtained (Give A1 for two components)
True for $n=k \Rightarrow$ True for $n=k+1$ (Hence true for all positive integers n)	A1 **8**	Stated or clearly implied *Dependent on previous 6 marks*