

**Proof By Induction (Inductive Sequences) Exam Questions****From OCR 4725****Q1, (Jan 2008, Q8)**

The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 1$  and  $u_{n+1} = u_n + 2n + 1$ .

- (i) Show that  $u_4 = 16$ . [2]
- (ii) Hence suggest an expression for  $u_n$ . [1]
- (iii) Use induction to prove that your answer to part (ii) is correct. [4]
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**Q2, (Jun 2009, Q10)**

The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 3$  and  $u_{n+1} = 3u_n - 2$ .

- (i) Find  $u_2$  and  $u_3$  and verify that  $\frac{1}{2}(u_4 - 1) = 27$ . [3]
- (ii) Hence suggest an expression for  $u_n$ . [2]
- (iii) Use induction to prove that your answer to part (ii) is correct. [5]
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**Q3, (Jan 2011, Q3)**

The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 2$ , and  $u_{n+1} = 2u_n - 1$  for  $n \geq 1$ . Prove by induction that  $u_n = 2^{n-1} + 1$ . [4]

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**Q4, (Jan 2013, Q10)**

The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 2$  and  $u_{n+1} = \frac{u_n}{1 + u_n}$  for  $n \geq 1$ .

- (i) Find  $u_2$  and  $u_3$ , and show that  $u_4 = \frac{2}{7}$ . [3]
- (ii) Hence suggest an expression for  $u_n$ . [2]
- (iii) Use induction to prove that your answer to part (ii) is correct. [5]
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**Q5, (Jun 2016, Q5)**

The sequence  $u_1, u_2, u_3, \dots$  is defined by  
$$u_1 = 5 \text{ and } u_{n+1} = 3u_n + 2 \text{ for } n \geq 1.$$

Prove by induction that  $u_n = 2 \times 3^n - 1$ . [4]

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**From OCR 4755**

**Q6, (Jan 2008, Q6)**

A sequence is defined by  $a_1 = 7$  and  $a_{k+1} = 7a_k - 3$ .

(i) Calculate the value of the third term,  $a_3$ . [2]

(ii) Prove by induction that  $a_n = \frac{(13 \times 7^{n-1}) + 1}{2}$ . [6]

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**Q7, (Jun 2010, Q6)**

A sequence is defined by  $u_1 = 2$  and  $u_{n+1} = \frac{u_n}{1 + u_n}$ .

(i) Calculate  $u_3$ . [2]

(ii) Prove by induction that  $u_n = \frac{2}{2n - 1}$ . [6]

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**Q8, (Jan 2011, Q3)**

A sequence is defined by  $u_1 = 5$  and  $u_{n+1} = u_n + 2^{n+1}$ . Prove by induction that  $u_n = 2^{n+1} + 1$ . [6]

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**Q9, (Jun 2012, Q6)**

A sequence is defined by  $a_1 = 1$  and  $a_{k+1} = 3(a_k + 1)$ .

(i) Calculate the value of the third term,  $a_3$ . [1]

(ii) Prove by induction that  $a_n = \frac{5 \times 3^{n-1} - 3}{2}$ . [6]

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