Proof By Induction (Inductive Sequences) Exam Questions

From OCR 4725

Q1, (Jan 2008, Q8)

(i) $u_2 = 4,\ u_3 = 9,\ u_4 = 16$	M1		Obtain next terms
	A1	2	All terms correct
(ii) $u_n = n^2$	B1	1	Sensible conjecture made
(iii)	B1		State that conjecture is true for $n = 1$ or 2
	M1		Find u_{n+1} in terms of n
	A1		Obtain $(n+1)^2$
	A1	4	Statement of Induction conclusion
		7	

Q2, (Jun 2009, Q10)

i) $u_2 = 7\ \ u_3 = 19$	M1		Attempt to find next 2 terms
	A1		Obtain correct answers
	A1	3	Show given result correctly
(ii) $u_n = 2(3^{n-1}) + 1$	M1		Expression involving a power of 3
	A1	2	Obtain correct answer
(iii)	B1ft		Verify result true when $n = 1$ or $n = 2$
$u_{n+1} = 3(2(3^{n-1})+1) - 2$	M1		Expression for u_{n+1} using recurrence relation
$u_{n+1} = 2(3^n) + 1$	A1		Correct unsimplified answer
	A1		Correct answer in correct form
	B1		Statement of induction conclusion
		5	
		10	

Q3, (Jan 2011, Q3)

B1*	Establish result true for $n = 1$ or 2
M1*	Use given result in recurrence relation in a relevant way
A1*	Obtain $2^n + 1$ correctly
depA1 **4**	Specific statement of induction conclusion
4	

Q4, (Jan 2013, Q10)

(i)		$\frac{2}{3},\ \frac{2}{5},\ \frac{2}{7}$	B1 B1 B1 **[3]**	B1 x 3, Obtain 3 correct values Justify **given** answer
(ii)		$\frac{2}{2n-1}$	M1 A1 **[2]**	Fraction, in terms of n, with correct numerator or denominator Obtain correct answer a.e.f.
(iii)		$\frac{2}{2(n+1)-1}$	B1ft M1 A1 A1 B1 **[5]**	Verify result true when $n = 1$, for their (ii), or n = 2, 3 or 4 Expression for u_{n+1} using recurrence relation in terms of n using their (ii) Correct unsimplified answer Correct answer in correct form Specific statement of induction conclusion, previous 4 marks must be earned, n=1 must be verified

Q5, (Jun 2016, Q5)

$3(2\times 3^n - 1) + 2$	B1	Show clearly result true for $n = 1$, accept $(u_1) = 2\times 3 - 1 = 5$
	M1	Substitute for u_n in recurrence relation
	A1	Establish correct result for u_{n+1} convincingly
	B1	Clear statement of induction conclusion, provided 1st 3 marks earned
	[4]	

From OCR 4755

Q6, (Jan 2008, Q6)

(i)	$a_2 = 7\times 7 - 3 = 46$	M1	Use of inductive definition
	$a_3 = 7\times 46 - 3 = 319$	A1 [2]	c.a.o.
(ii)	When n = 1, $\frac{13\times 7^0 + 1}{2} = 7$, so true for n = 1	B1	Correct use of part (i) (may be implied)
	Assume true for $n = k$ $a_k = \frac{13\times 7^{k-1}+1}{2}$	E1	Assuming true for k
	$\Rightarrow a_{k+1} = 7\times\frac{13\times 7^{k-1}+1}{2} - 3$	M1	Attempt to use $a_{k+1} = 7a_k - 3$
	$= \frac{13\times 7^k + 7}{2} - 3$ $= \frac{13\times 7^k + 7 - 6}{2}$ $= \frac{13\times 7^k + 1}{2}$	A1	Correct simplification
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1.	E1	Dependent on A1 and previous E1
	Since it is true for k = 1, it is true for k = 1, 2, 3 and so true for all positive integers.	E1 [6]	Dependent on B1 and previous E1

Q7, (Jun 2010, Q6)

6(i)	$u_2 = \frac{2}{1+2} = \frac{2}{3}, u_3 = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$	M1 A1 **[2]**	Use of inductive definition c.a.o.
6(ii)	When $n = 1$, $\frac{2}{2\times1-1} = 2$, so true for $n = 1$	B1	Showing use of $u_n = \frac{2}{2n-1}$
	Assume $u_k = \frac{2}{2k-1}$	E1	Assuming true for k
	$\Rightarrow u_{k+1} = \frac{\frac{2}{2k-1}}{1+\frac{2}{2k-1}}$	M1	u_{k+1}
	$= \frac{\frac{2}{2k-1}}{\frac{2k-1+2}{2k-1}} = \frac{2}{2k+1}$	A1	Correct simplification
	$= \frac{2}{2(k+1)-1}$		
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is also true for $k + 1$.	E1	Dependent on A1 and previous E1
	Since it is true for $k = 1$, it is true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1

Q8, (Jan 2011, Q3)

3	B1*	Establish result true for $n = 1$ or 2
	M1*	Use given result in recurrence relation in a relevant way
	A1*	Obtain $2^n + 1$ correctly
	depA1 **4**	Specific statement of induction conclusion
	4	

Q9, (Jun 2012, Q6)

(i)	$a_2 = 3\times2 = 6, a_3 = 3\times7 = 21$	B1 **[1]**	cao
(ii)	When $n = 1$, $\frac{5\times3^0-3}{2} = 1$, so true for $n = 1$	B1	Showing use of $a_n = \frac{5\times3^{n-1}-3}{2}$
	Assume $a_k = \frac{5\times3^{k-1}-3}{2}$	E1	Assuming true for $n = k$
	$\Rightarrow a_{k+1} = 3\left(\frac{5\times3^{k-1}-3}{2}+1\right)$	M1	a_{k+1}, using a_k and attempting to simplify
	$= \frac{5\times3^k-9}{2}+3 = \frac{5\times3^k-9+6}{2}$		
	$= \frac{5\times3^k-3}{2} = \frac{5\times3^{(k+1)-1}-3}{2}$	A1	Correct simplification to left hand expression.
	But this is the given result with $k+1$ replacing k.		May be identified with a 'target' expression using $n = k+1$
	Therefore if it is true for $n = k$ it is also true for $n = k+1$.	E1	Dependent on A1 and previous E1
	Since it is true for $n = 1$, it is true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1