Proof By Induction (Divisibility) Exam Questions (From OCR 4725 unless otherwise stated)

Q1, (OCR 4725, Jan 2007, Q6)

The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = n^2 + 3n$, for all positive integers n.

(i) Show that $u_{n+1} - u_n = 2n + 4$. [3]

(ii) Hence prove by induction that each term of the sequence is divisible by 2. [5]

Q2, (OCR 4725, Jan 2009, Q7)

It is given that $u_n = 13^n + 6^{n-1}$, where n is a positive integer.

(i) Show that $u_n + u_{n+1} = 14 \times 13^n + 7 \times 6^{n-1}$. **[3]**

(ii) Prove by induction that u_n is a multiple of 7. **[4]**

Q3, (OCR 4725, Jun 2014, Q10)

The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5^n + 2^{n-1}$.

(i) Find u_1, u_2 and u_3. **[2]**

(ii) Hence suggest a positive integer, other than 1, which divides exactly into every term of the sequence. **[1]**

(iii) By considering $u_{n+1} + u_n$, prove by induction that your suggestion in part **(ii)** is correct. **[5]**

Q4, (Edexcel 6667, Jun 2009, Q8)

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 5^n + 8n + 3 \text{ is divisible by } 4,$$

(7)

Q5, (Edexcel 6667, Jun 2010, Q7)

$$f(n) = 2^n + 6^n$$

(a) Show that $f(k+1) = 6f(k) - 4(2^k)$.

(3)

(b) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 8.

(4)

Q6, (Edexcel 6667, Jun 2012, Q10)

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1} \text{ is divisible by } 5.$$

(6)

Q7, (Edexcel 6667A, Jan 2014, Q10)

(i) A sequence of numbers $u_1, u_2, u_3, \ldots$, is defined by

$$u_{n+1} = 5u_n + 3, \quad u_1 = 3$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{3}{4}(5^n - 1)$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 5(5^n) - 4n - 5 \text{ is divisible by } 16.$$

(6)

Q8, (Edexcel 6667, Jun 2014, Q9)

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 8^n - 2^n$$

is divisible by 6

(6)
