Solving Simultaneous Equations with Matrices (From OCR 4725)

Q1, (Jun 2005, Q7)

The matrix \( \mathbf{B} \) is given by
\[
\begin{pmatrix}
  a & 1 & 3 \\
  2 & 1 & -1 \\
  0 & 1 & 2
\end{pmatrix}.
\]

(i) Given that \( \mathbf{B} \) is singular, show that \( a = -\frac{2}{3} \). [3]

(ii) Given instead that \( \mathbf{B} \) is non-singular, find the inverse matrix \( \mathbf{B}^{-1} \). [4]

(iii) Hence, or otherwise, solve the equations
\[-x + y + 3z = 1, \quad 2x + y - z = 4, \quad y + 2z = -1.\] [3]

Q2, (Jun 2006, Q8)

The matrix \( \mathbf{M} \) is given by
\[
\begin{pmatrix}
  a & 4 & 2 \\
  1 & a & 0 \\
  1 & 2 & 1
\end{pmatrix}.
\]

(i) Find, in terms of \( a \), the determinant of \( \mathbf{M} \). [3]

(ii) Hence find the values of \( a \) for which \( \mathbf{M} \) is singular. [3]

Q3, (Jan 2008, Q7)

The matrix \( \mathbf{A} \) is given by
\[
\begin{pmatrix}
  a & 3 \\
  -2 & 1
\end{pmatrix}.
\]

(i) Given that \( \mathbf{A} \) is singular, find \( a \). [2]

(ii) Given instead that \( \mathbf{A} \) is non-singular, find \( \mathbf{A}^{-1} \) and hence solve the simultaneous equations
\[ax + 3y = 1, \quad -2x + y = -1.\] [5]

Q4, (Jan 2009, Q5)

By using the determinant of an appropriate matrix, or otherwise, find the value of \( k \) for which the simultaneous equations
\[2x - y + z = 7, \quad 3y + z = 4, \quad x + ky + kz = 5,\]
do not have a unique solution for \( x, y \) and \( z \). [5]
The matrix \( \mathbf{A} \) is given by

\[
\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}.
\]

(i) Find the value of \( a \) for which \( \mathbf{A} \) is singular. [5]

(ii) Given that \( \mathbf{A} \) is non-singular, find \( \mathbf{A}^{-1} \) and hence solve the equations

\[
\begin{align*}
ax + 2y + z &= 1, \\
x + 3y + 2z &= 2, \\
4x + y + z &= 3.
\end{align*}
\] [7]