Solving Simultaneous Equations with Matrices (From OCR 4725)

Q1, (Jun 2005, Q7)

i/ $|B| = a(3) - 2(-1) + 0 = 0 \Rightarrow 3a = -2 \Rightarrow a = -\frac{2}{3}$

ii/ $m_{11} = 3 \quad m_{12} = 4 \quad m_{13} = 2$

$m_{21} = -1 \quad m_{22} = 2a \quad m_{23} = a$

$m_{31} = -4 \quad m_{32} = -a-6 \quad m_{33} = a-2$

$\Rightarrow M = \begin{pmatrix} 3 & 4 & 2 \\ -1 & 2a & a \\ -4 & -a-6 & a-2 \end{pmatrix}$

$\Rightarrow C = \begin{pmatrix} 3 & -4 & 2 \\ 1 & 2a & -a \\ -4 & a+6 & a-2 \end{pmatrix} \quad \Rightarrow C^T = \begin{pmatrix} 3 & 1 & -4 \\ -4 & 2a & a+6 \\ 2 & -a & a-2 \end{pmatrix}$

$|B| = 3a + 2 \quad \Rightarrow \quad B^{-1} = \frac{1}{3a+2}\begin{pmatrix} 3 & 1 & -4 \\ -4 & 2a & a+6 \\ 2 & -a & a-2 \end{pmatrix}$

iii/ $\begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$

$\begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{pmatrix}^{-1} = \frac{1}{3(-1)+2}\begin{pmatrix} 3 & 1 & -4 \\ -4 & 2(-1) & -1+6 \\ 2 & -(-1) & -1-2 \end{pmatrix} = -1\begin{pmatrix} 3 & 1 & -4 \\ -4 & -2 & 5 \\ 2 & 1 & -3 \end{pmatrix}$

$= \begin{pmatrix} -3 & -1 & 4 \\ 4 & 2 & -5 \\ -2 & -1 & 3 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 & -1 & 4 \\ 4 & 2 & -5 \\ -2 & -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \\ -3 \end{pmatrix} \quad \therefore x = -1,\ y = 17,\ z = -9$

Q2, (Jun 2006, Q8)

(i)			
$a\begin{bmatrix} a & 0 \\ 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & a \\ 1 & 2 \end{bmatrix}$	M1		Correct expansion process shown
	A1		Obtain correct unsimplified expression
$a^2 - 2a$	A1	3	Obtain correct answer
(ii)	M1		Solve their det **M** = 0
$a = 0$ or $a = 2$	A1A1ft	3	Obtain correct answers

Q3, (Jan 2008, Q7)

(i)			
$a = -6$	M1		Use det **A** = 0
	A1	2	Obtain correct answer
(ii) $\mathbf{A}^{-1} = \frac{1}{a+6}\begin{pmatrix} 1 & -3 \\ 2 & a \end{pmatrix}$	B1		Both diagonals correct
	B1ft		Divide by det **A**
	M1		Premultiply column by $\mathbf{A}^{-1}$, no other method
$x = \frac{4}{a+6}, y = \frac{2-a}{a+6}$	A1ft		Obtain correct answers from their $\mathbf{A}^{-1}$
	A1ft	5	
		7	

Q4, (Jan 2009, Q5)

Either	M1		Consider determinant of coefficients of LHS
	M1		Sensible attempt at evaluating any 3×3 det
$4k - 4$	A1		Obtain correct answer a.e.f. unsimplified
	M1		Equate det to 0
$k = 1$	A1ft	5	Obtain $k = 1$, ft provided all M's awarded
Or	M1		Eliminate either x or y
	A1		Obtain correct equation
	M1		Eliminate 2nd variable
	A1		Obtain correct linear equation
	A1		Deduce that $k = 1$
		5	

Q5, (Jun 2013, Q10)

(i)			
	M1		Show correct expansion process for 3 × 3
	M1		Correct evaluation of any 2 × 2
$a + 3$	A1		Obtain correct answer
	M1		Use det **A** = 0
$a = -3$	A1FT		Obtain correct answer from their det **A**
	[5]		
(ii)			
$\frac{1}{a+3}\begin{pmatrix} 1 & -1 & 1 \\ 7 & a-4 & 1-2a \\ -11 & 8-a & 3a-2 \end{pmatrix}$	M1		Show correct processes for adjoint entries
	A1		Obtain at least 4 correct entries in adjoint
	A1		Obtain completely correct adjoint
	B1		Divide adjoint by their det **A**
$\frac{1}{a+3}\begin{pmatrix} 2 \\ 2-4a \\ 7a-1 \end{pmatrix}$	M1		Pre-multiply column matrix by their $\mathbf{A}^{-1}$
	A2		Obtain correct answer, A1 for 1 element correct
	[7]		