## Matrix Operations, Determinants and Inverses

**Q1, (Jan 2006, Q1)**

You are given that $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}$.

**(i)** Calculate, where possible, $2\mathbf{B}$, $\mathbf{A} + \mathbf{C}$, $\mathbf{CA}$ and $\mathbf{A} - \mathbf{B}$. [5]

**(ii)** Show that matrix multiplication is not commutative. [2]

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**Q2, (Jun 2007, Q1i)**

You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$.

**(i)** Find the inverse of $\mathbf{M}$. [2]

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**Q3, (Jan 2007, Q9)**

Matrices $\mathbf{M}$ and $\mathbf{N}$ are given by $\mathbf{M} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix}$.

**(i)** Find $\mathbf{M}^{-1}$ and $\mathbf{N}^{-1}$. [3]

**(ii)** Find $\mathbf{MN}$ and $(\mathbf{MN})^{-1}$. Verify that $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$. [6]

**(iii)** The result $(\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ is true for any two $2 \times 2$, non-singular matrices $\mathbf{P}$ and $\mathbf{Q}$.

The first two lines of a proof of this general result are given below. Beginning with these two lines, complete the general proof.

$$(\mathbf{PQ})^{-1}\mathbf{PQ} = \mathbf{I}$$

$$\Rightarrow \quad (\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1} = \mathbf{IQ}^{-1}$$ [4]

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**Q4, (Jun 2009, Q1)**

**(i)** Find the inverse of the matrix $\mathbf{M} = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$. **[2]**

**(ii)** Use this inverse to solve the simultaneous equations

$$4x - y = 49,$$
$$3x + 2y = 100,$$

showing your working clearly. **[3]**

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**Q5, (Jun 2015, Q1)**

Given that $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, where $\mathbf{M} = \begin{pmatrix} 4 & -3 \\ 8 & 21 \end{pmatrix}$, find $x$ and $y$. **[6]**

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**Q6, (Jun 2010, Q2)**

You are given that $\mathbf{M} = \begin{pmatrix} 2 & -5 \\ 3 & 7 \end{pmatrix}$.

$\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \end{pmatrix}$ represents two simultaneous equations.

**(i)** Write down these two equations. **[2]**

**(ii)** Find $\mathbf{M}^{-1}$ and use it to solve the equations. **[4]**

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**Q7, (Jun 2011, Q9)**

The simultaneous equations

$$2x - \phantom{k}y = 1$$
$$3x + ky = b$$

are represented by the matrix equation $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ b \end{pmatrix}$.

**(i)** Write down the matrix $\mathbf{M}$. **[2]**

**(ii)** State the value of $k$ for which $\mathbf{M}^{-1}$ does not exist and find $\mathbf{M}^{-1}$ in terms of $k$ when $\mathbf{M}^{-1}$ exists.

Use $\mathbf{M}^{-1}$ to solve the simultaneous equations when $k = 5$ and $b = 21$. **[7]**

**(iii)** What can you say about the solutions of the equations when $k = -\frac{3}{2}$? **[1]**

**(iv)** The two equations can be interpreted as representing two lines in the $x$-$y$ plane. Describe the relationship between these two lines

($A$) when $k = 5$ and $b = 21$,

($B$) when $k = -\frac{3}{2}$ and $b = 1$,

($C$) when $k = -\frac{3}{2}$ and $b = \frac{3}{2}$. **[3]**

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