Matrices – Invariant Points and Lines Exam Questions (From OCR 4755)

Q1, (Jun 2006, Q5)

(i) The matrix $\mathbf{S} = \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$ represents a transformation.

(*A*) Show that the point (1, 1) is invariant under this transformation. [1]

(*B*) Calculate $\mathbf{S}^{-1}$. [2]

(*C*) Verify that (1, 1) is also invariant under the transformation represented by $\mathbf{S}^{-1}$. [1]

(ii) Part **(i)** may be generalised as follows.

If (x, y) is an invariant point under a transformation represented by the non-singular matrix $\mathbf{T}$, it is also invariant under the transformation represented by $\mathbf{T}^{-1}$.

Starting with $\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$, or otherwise, prove this result. [2]

Q2, (Jun 2008, Q3)

Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}$. [3]

Q3, (Jan 2004, Q9)

You are given the matrix $\mathbf{M} = \begin{pmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix}$.

(i) Calculate $\mathbf{M}^2$. [1]

You are now given that the matrix $\mathbf{M}$ represents a reflection in a line through the origin.

(ii) Explain how your answer to part **(i)** relates to this information. [1]

(iii) By investigating the invariant points of the reflection, find the equation of the mirror line. [3]

(iv) Describe fully the transformation represented by the matrix $\mathbf{P} = \begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$. [2]

(v) A composite transformation is formed by the transformation represented by $\mathbf{P}$ followed by the transformation represented by $\mathbf{M}$. Find the single matrix that represents this composite transformation. [2]

(vi) The composite transformation described in part **(v)** is equivalent to a single reflection. What is the equation of the mirror line of this reflection? [1]

Q4, (Jun 2005, Q3)

Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$. [3]

Q5, (OCR 2604, Jun 2001, Q4b)

The matrix $\begin{pmatrix} 4 & 2 \\ 6 & k \end{pmatrix}$, where k is a constant, defines a transformation in the (x, y)-plane.

(i) Find the set of invariant points of the transformation

(A) when $k = 4$,

(B) when $k = 5$. [5]

(ii) When $k = 5$, verify that $y = 2x - 3$ is an invariant line of the transformation. [4]

Q6 (OCR 2604, Jun 2002, Q4b)

M is the transformation of the plane defined by the matrix $\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}$.

(i) Show that $y = 3x$ is an invariant line of the transformation **M**, and find the equation of the other invariant line. [5]

P is the transformation defined by the matrix $\begin{pmatrix} 5 & -1 \\ -8 & 2 \end{pmatrix}$.

The line L is the image of the line $y = 3x$ under the transformation **P**.

(ii) Find the equation of L. [2]

The transformation **Q** is the inverse of **P**.

(iii) State the image of the line L under the transformation **Q**. [1]

(iv) Find the matrix corresponding to **Q**. [2]

The transformation **R** is **Q** followed by **M** followed by **P**.

(v) Show that L is an invariant line of the transformation **R**. [2]

(vi) Find the matrix corresponding to **R**. [3]