Matrices – Invariant Points and Lines Exam Questions (From OCR 4755)

Q1, (Jun 2006, Q5)
(i) The matrix \( S = \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix} \) represents a transformation.

(A) Show that the point \((1, 1)\) is invariant under this transformation. \([1]\)

(B) Calculate \( S^{-1} \). \([2]\)

(C) Verify that \((1, 1)\) is also invariant under the transformation represented by \( S^{-1} \). \([1]\)

(ii) Part (i) may be generalised as follows.

If \((x, y)\) is an invariant point under a transformation represented by the non-singular matrix \( T \), it is also invariant under the transformation represented by \( T^{-1} \).

Starting with \( T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \), or otherwise, prove this result. \([2]\)

Q2, (Jun 2008, Q3)
Find the equation of the line of invariant points under the transformation given by the matrix
\( M = \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix} \). \([3]\)

Q3, (Jan 2004, Q9)
You are given the matrix \( M = \begin{pmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix} \).

(i) Calculate \( M^2 \). \([1]\)

You are now given that the matrix \( M \) represents a reflection in a line through the origin.

(ii) Explain how your answer to part (i) relates to this information. \([1]\)

(iii) By investigating the invariant points of the reflection, find the equation of the mirror line. \([3]\)

(iv) Describe fully the transformation represented by the matrix \( P = \begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix} \). \([2]\)

(v) A composite transformation is formed by the transformation represented by \( P \) followed by the transformation represented by \( M \). Find the single matrix that represents this composite transformation. \([2]\)

(vi) The composite transformation described in part (v) is equivalent to a single reflection. What is the equation of the mirror line of this reflection? \([1]\)
Q4, (Jun 2005, Q3)

Find the equation of the line of invariant points under the transformation given by the matrix 

\[ M = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}. \]

Q5, (OCR 2604, Jun 2001, Q4b)

The matrix \( \begin{pmatrix} 4 & 2 \\ 6 & k \end{pmatrix} \), where \( k \) is a constant, defines a transformation in the \((x, y)\)-plane.

(i) Find the set of invariant points of the transformation

(A) when \( k = 4 \),

(B) when \( k = 5 \).

(ii) When \( k = 5 \), verify that \( y = 2x - 3 \) is an invariant line of the transformation.

Q6 (OCR 2604, Jun 2002, Q4b)

\[ M \] is the transformation of the plane defined by the matrix \( \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \).

(i) Show that \( y = 3x \) is an invariant line of the transformation \( M \), and find the equation of the other invariant line.

\[ P \] is the transformation defined by the matrix \( \begin{pmatrix} 5 & -1 \\ -8 & 2 \end{pmatrix} \).

The line \( L \) is the image of the line \( y = 3x \) under the transformation \( P \).

(ii) Find the equation of \( L \).

The transformation \( Q \) is the inverse of \( P \).

(iii) State the image of the line \( L \) under the transformation \( Q \).

(iv) Find the matrix corresponding to \( Q \).

The transformation \( R \) is \( Q \) followed by \( M \) followed by \( P \).

(v) Show that \( L \) is an invariant line of the transformation \( R \).

(vi) Find the matrix corresponding to \( R \).