Matrices – Invariant Points and Lines Exam Questions (From OCR 4755)

Q1, (Jun 2006, Q5)

(i)	$\begin{pmatrix}-1 & 2\\ -3 & 4\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$	B1	
	$\mathbf{S}^{-1}=\frac{1}{2}\begin{pmatrix}4 & -2\\ 3 & -1\end{pmatrix}$	M1,	Attempt to divide by determinant and manipulate contents
		A1	Correct
	$\frac{1}{2}\begin{pmatrix}4 & -2\\ 3 & -1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$	E1	
		[4]	
ii)	$\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$		
	$\Rightarrow \mathbf{T}^{-1}\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix}=\mathbf{T}^{-1}\begin{pmatrix}x\\y\end{pmatrix}$	M1	Pre-multiply by $\mathbf{T}^{-1}$
	$\Rightarrow \begin{pmatrix}x\\y\end{pmatrix}=\mathbf{T}^{-1}\begin{pmatrix}x\\y\end{pmatrix}$	A1	Invariance shown
		[2]	

Q2, (Jun 2008, Q3)

$\begin{pmatrix}-1 & -1\\ 2 & 2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$	M1	For $\begin{pmatrix}-1 & -1\\ 2 & 2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$
$\Rightarrow -x-y=x,\ 2x+2y=y$	M1	
$\Rightarrow y=-2x$	B1	
	[3]	

Q3, (Jan 2005, Q9)

(i)	$\mathbf{M}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$	B1 [1]	.
(ii)	$\mathbf{M}^2$ gives the identity because a reflection, followed by a second reflection in the same mirror line will get you back where you started OR reflection matrices are self-inverse.	E1 [1]	
(iii)	$\begin{pmatrix} 0.8 & 0.6 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow 0.8x + 0.6y = x$ and $0.6x - 0.8y = y$ Both of these lead to $y = \frac{1}{3}x$ as the equation of the mirror line.	M1 A1 A1 [3]	Give both marks for either equation or for a correct geometrical argument
(iv)	Rotation, centre origin, 36.9° anticlockwise.	B1, B1 [2]	One for rotation and centre, one for angle and sense. Accept 323.1° clockwise or radian equivalents (0.644 or 5.64).
(v)	$\mathbf{MP} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1, A1 [2]	
(vi)	$y = 0$	B1 [1]	

Q4, (Jun 2005, Q3)

$\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow x = 3x - y,\ y = 2x$ $\Rightarrow y = 2x$	M1 A1 A1 [3]	M1 for $\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ (allow if implied) $\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} k \\ mk \end{pmatrix} = \begin{pmatrix} K \\ mK \end{pmatrix}$ can lead to full marks if correctly used. Lose second A1 if answer includes two lines

Q5, (OCR 2604, Jun 2001, Q4b)

)(i)	(*A*) $\begin{pmatrix}4 & 2\\6 & 4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4x+2y\\6x+4y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$	M1		
	The only invariant point is $(0, 0)$	A2		
	(*B*) $\begin{pmatrix}4 & 2\\6 & 5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4x+2y\\6x+5y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$	M1		
	Set of invariant points is the line $y=-\frac{3}{2}x$	A1	5	
(ii)	$\begin{pmatrix}4 & 2\\6 & 5\end{pmatrix}\begin{pmatrix}t\\2t-3\end{pmatrix}$	M1		Or finding images of *two* points on $y=2x-3$
	$=\begin{pmatrix}8t-6\\16t-15\end{pmatrix}$	A1		Or images of two points correct
		M1		Checking image(s) in $y'=2x'-3$ *Or* finding equation of image line
	$2(8t-6)-3=16t-15$, so the line $y=2x-3$ is invariant	A1	4	
	OR The line $y=mx+c$ is invariant if $6x+5(mx+c)=m[4x+2(mx+c)]+c$ M2			
	which is satisfied by $m=2$, $c=-3$ A2			Properly shown

(b)(i)	$\begin{pmatrix}-1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}t\\ mt\end{pmatrix}=\begin{pmatrix}-t+2mt\\ 3t+4mt\end{pmatrix}$ The line $y=mx$ is invariant if $3t+4mt=m(-t+2mt)$ $2m^2-5m-3=0$ $(m-3)(2m+1)=0$ $m=3,\ -\frac{1}{2}$ The other invariant line is $y=-\frac{1}{2}x$	M1 A1 M1 B1 A1 **5**	For LHS (or equivalent) For $m=3$. Can be given for $\begin{pmatrix}-1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}1\\ 3\end{pmatrix}=\begin{pmatrix}5\\ 15\end{pmatrix}$, also on $y=3x$
(ii)	$\begin{pmatrix}5 & -1\\ -8 & 2\end{pmatrix}\begin{pmatrix}t\\ 3t\end{pmatrix}=\begin{pmatrix}2t\\ -2t\end{pmatrix}$ Equation of L is $y=-x$	M1 A1 **2**	Any one non-zero point on $y=3x$ is sufficient
(iii)	The line $y=3x$	B1 **1**	
(iv)	$\mathbf{Q}=\mathbf{P}^{-1}=\frac{1}{2}\begin{pmatrix}2 & 1\\ 8 & 5\end{pmatrix}$	B1B1 **2**	For $\frac{1}{2}$ and $\begin{pmatrix}2 & 1\\ 8 & 5\end{pmatrix}$
(v)	**Q** transforms L to $y=3x$ **M** transforms $y=3x$ to $y=3x$ **P** transforms $y=3x$ to L Hence **R** transforms L to L	M1 A1 **2**	Considering at least two successive transformations of L
	OR, if done after **(vi)**, $\begin{pmatrix}16 & 11\\ -18 & -13\end{pmatrix}\begin{pmatrix}t\\ -t\end{pmatrix}=\begin{pmatrix}5t\\ -5t\end{pmatrix}$ M1 which is also on L A1		
(vi)	$\mathbf{R}=\mathbf{PMQ}=\begin{pmatrix}5 & -1\\ -8 & 2\end{pmatrix}\begin{pmatrix}-1 & 2\\ 3 & 4\end{pmatrix}\frac{1}{2}\begin{pmatrix}2 & 1\\ 8 & 5\end{pmatrix}$ $=\begin{pmatrix}16 & 11\\ -18 & -13\end{pmatrix}$	M1A1 ft A1 cao **3**	Give M1A0 if order wrong