3x3 Matrix Determinants and Inverses (From OCR 4755)

Q1, (Jun 2008, Q5)

(i)	$\mathbf{AB} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$	B3 [3]	Minus 1 each error to minimum of 0
(ii)	$\mathbf{A}^{-1} = \frac{1}{7}\begin{pmatrix} -1 & 0 & 2 \\ 14 & -14 & 7 \\ -5 & 7 & -4 \end{pmatrix}$	M1 A1 [2]	Use of B c.a.o.

Q2, (Jan 2009, Q10)

(i)	$\alpha = 3\times -5 + 4\times 11 + -1\times 29 = 0$ $\beta = -2\times -7 + 7\times(5+k) + -3\times 7 = 28 + 7k$	B1 M1 A1 [3]	 Attempt at row 3 x column 3
(ii)	$\mathbf{AB} = \begin{pmatrix} 42 & 0 & 0 \\ 0 & 42 & 0 \\ 0 & 0 & 42 \end{pmatrix}$	B2 [2]	Minus 1 each error to min of 0
(iii)	$\mathbf{A}^{-1} = \frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}$	M1 B1 A1 [3]	Use of **B** $\frac{1}{42}$ Correct inverse, allow decimals to 3 sf
(iv)	$\frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}\begin{pmatrix} 1 \\ -9 \\ 26 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $= \frac{1}{42}\begin{pmatrix} -126 \\ 84 \\ -84 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -2 \end{pmatrix}$ $x = -3,\ y = 2,\ z = -2$	M1 A3 [4]	Attempt to pre-multiply by $\mathbf{A}^{-1}$ SC B2 for Gaussian elimination with 3 correct solutions, -1 each error to min of 0 Minus 1 each error

Q3, (Jan 2010, Q4)

$\mathbf{MM}^{-1}=\frac{1}{k}\begin{pmatrix}4&0&1\\-6&1&1\\5&2&5\end{pmatrix}\begin{pmatrix}-3&-2&1\\-35&-15&10\\17&8&-4\end{pmatrix}$	M1	Attempt to consider $\mathbf{MM}^{-1}$ or $\mathbf{M}^{-1}\mathbf{M}$ (may be implied)
$=\frac{1}{k}\begin{pmatrix}5&0&0\\0&5&0\\0&0&5\end{pmatrix}\Rightarrow k=5$	A1 [2]	c.a.o.
$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{5}\begin{pmatrix}-3&-2&1\\-35&-15&10\\17&8&-4\end{pmatrix}\begin{pmatrix}9\\32\\81\end{pmatrix}$	M1	Attempt to pre-multiply by $\mathbf{M}^{-1}$
	M1	Attempt to multiply matrices
$\frac{1}{5}\begin{pmatrix}-3&-2&1\\-35&-15&10\\17&8&-4\end{pmatrix}\begin{pmatrix}9\\32\\81\end{pmatrix}=\frac{1}{5}\begin{pmatrix}-10\\15\\85\end{pmatrix}$	A1	Correct
$\Rightarrow x=-2,\ y=3,\ z=17$	A1 [4]	All 3 correct s.c. B1 if matrices not used

Q4, (Jun 2013, Q3)

(i)	$-2-4p=0$	M1	Any valid row x column leadi
	$\Rightarrow p=-\frac{1}{2}$	B1 [2]	
(ii)	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\mathbf{N}^{-1}\begin{pmatrix}-39\\5\\22\end{pmatrix}$	M1	Attempt to use $\mathbf{N}^{-1}$
	$=\begin{pmatrix}1&0&2\\2&1&3\\\frac{-7}{2}&\frac{-1}{2}&-6\end{pmatrix}\begin{pmatrix}-39\\5\\22\end{pmatrix}$	M1	Attempt to multiply matrices
	$=\begin{pmatrix}5\\-7\\2\end{pmatrix}$	A1	One element correct
		A1 [4]	All 3 correct. FT their p

Q5, (Jan 2011, Q9)

(i)	$\mathbf{AB}=\begin{pmatrix}-2&1&-5\\3&a&1\\1&-1&2\end{pmatrix}\begin{pmatrix}2a+1&3&1+5a\\-5&1&-13\\-3-a&-1&-2a-3\end{pmatrix}$		
	$=\begin{pmatrix}-4a-2-5+15+5a&0&0\\0&9+a-1&0\\0&0&1+5a+13-4a-6\end{pmatrix}$	M1	Attempt to find **AB** with some justification of at least two leading diagonal terms and any other
	$=\begin{pmatrix}8+a&0&0\\0&8+a&0\\0&0&8+a\end{pmatrix}$	A1	Correct
	$=(8+a)\mathbf{I}$	B1	Relating correct diagonal matrix to **I**
		[3]	
(ii)	$\mathbf{A}^{-1}$ does not exist for $a=-8$	B1	
	$\mathbf{A}^{-1}=\frac{1}{8+a}\begin{pmatrix}2a+1&3&1+5a\\-5&1&-13\\-3-a&-1&-2a-3\end{pmatrix}$	M1 A1	$k\mathbf{B}$, k not equal to 1 Correct $\mathbf{A}^{-1}$ as shown
		[3]	
(iii)	$\mathbf{A}^{-1}=\frac{1}{12}\begin{pmatrix}9&3&21\\-5&1&-13\\-7&-1&-11\end{pmatrix}$	B1	
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{12}\begin{pmatrix}9&3&21\\-5&1&-13\\-7&-1&-11\end{pmatrix}\begin{pmatrix}-55\\-9\\26\end{pmatrix}=\begin{pmatrix}2\\-6\\9\end{pmatrix}$	M1 A3 **[5]**	Correct use of their $\mathbf{A}^{-1}$ x, y and z cao, -1 each error
(iv)	There is no unique solution.	B1 **[1]**	

Q6, (Jun 2014, Q9)

(i)	$\beta=(-1)(3\alpha-1)+5\alpha+(-1)(2\alpha+1)$ $=-3\alpha+1+5\alpha-2\alpha-1=0$	M1 A1 **[2]**	multiply second row of **A** with first column of **B** Correct
(ii)	$\gamma=(1)(3\alpha-1)+15+(-1)(2\alpha+1)$ $=\alpha+13$	M1 A1 **[2]**	Attempt to multiply relevant row of **A** with relevant column of **B.** Condone use of **BA** instead Correct
(iii)	When $\alpha=2,\ \gamma=15$ $\mathbf{A}^{-1}=\frac{1}{15}\begin{pmatrix}5&-8&-1\\5&1&2\\5&-5&5\end{pmatrix}$ $\mathbf{A}^{-1}$ does not exist when $\alpha=-13$	M1 A1 B1ft **[3]**	Multiplication of **B** by $\frac{1}{\text{their }\gamma}$, $(\gamma\neq 1)$ using $\alpha=2$ in both Correct elements in matrix and correct γ. ft their $\gamma=0$. Condone "$\alpha\neq-13$"
(iv)	$\frac{1}{15}\begin{pmatrix}5&-8&-1\\5&1&2\\5&-5&5\end{pmatrix}\begin{pmatrix}25\\11\\-23\end{pmatrix}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ $=\frac{1}{15}\begin{pmatrix}60\\90\\-45\end{pmatrix}=\begin{pmatrix}4\\6\\-3\end{pmatrix}$ $\Rightarrow x=4,\ y=6,\ z=-3$	M1 B1 A3 **[5]**	Set-up of pre-multiplication by their 3x3 $\mathbf{A}^{-1}$, or by **B** (using $\alpha=2$) $(60\ \ 90\ \ -45)'$ soi need not be fully evaluated cao A1 for each explicit identification of x, y, z in a vector or a list. (-1 unidentified) Answers only or solution by other method, M0A0

Q7, (Jun 2016, Q3)

(i)	Either $\mu = (2)(34) + (5)(-5) + (-1)(18) = 25$ Or $\mu = (-1)(-14) + (4)(5) + (3)(-3)$ $-19\lambda + (6)(5) + (-4)(-13) = 25$ Or $34\lambda - 102 = 0$ Or $-14\lambda + 42 = 0$	M1 M1 A1 A1 **[4]**
(ii)	$\mathbf{B}^{-1} = \frac{1}{25}\begin{pmatrix} 3 & 6 & -4 \\ 2 & 5 & -1 \\ -1 & 4 & 3 \end{pmatrix}$	M1 B1 **[2]**