Basic Complex Number Operations Exam Questions (From OCR 4725)

Q1, (Jun 2005, Q3)
The complex numbers 2 + 3i and 4 – i are denoted by z and w respectively. Express each of the following in the form x + iy, showing clearly how you obtain your answers.

(i) \( z + 5w \), [2]
(ii) \( z^*w \), where \( z^* \) is the complex conjugate of z, [3]
(iii) \( \frac{1}{w} \). [2]

Q2, (Jun 2006, Q5)
The complex numbers 3 – 2i and 2 + i are denoted by z and w respectively. Find, giving your answers in the form x + iy and showing clearly how you obtain these answers,

(i) \( 2z – 3w \), [2]
(ii) \( (iz)^2 \), [3]
(iii) \( \frac{z}{w} \). [3]

Q3, (Jan 2009, Q1)
Express \( \frac{2 + 3i}{5 - i} \) in the form x + iy, showing clearly how you obtain your answer. [4]

Q4, (Jan 2011, Q2)
The complex numbers z and w are given by \( z = 4 + 3i \) and \( w = 6 - i \). Giving your answers in the form x + iy and showing clearly how you obtain them, find

(i) \( 3z - 4w \), [2]
(ii) \( \frac{z^*}{w} \). [4]

Q5, (Jun 2011, Q9)
One root of the quadratic equation \( x^2 + ax + b = 0 \), where \( a \) and \( b \) are real, is 16 – 30i.

(i) Write down the other root of the quadratic equation. [1]
(ii) Find the values of \( a \) and \( b \). [4]
(iii) Use an algebraic method to solve the quartic equation \( y^4 + ay^2 + b = 0 \). [7]

Q6, (Jun 2013, Q1)
The complex number \( 3 + ai \), where \( a \) is real, is denoted by \( z \). Given that \( \arg z = \frac{1}{6} \pi \), find the value of \( a \) and hence find \( |z| \) and \( z^* - 3 \). [6]

Q7, (Jun 2015, Q1)
The complex number \( x + iy \) is denoted by \( z \). Express \( 3z^* - |z|^2 \) in terms of \( x \) and \( y \). [3]