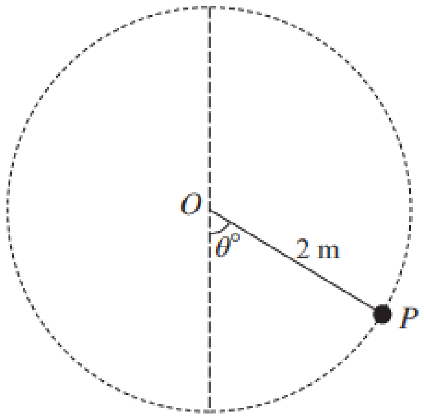
Motion in a Vertical Circle (From OCR 4730)

Q1, (Jan 2008, Q6)

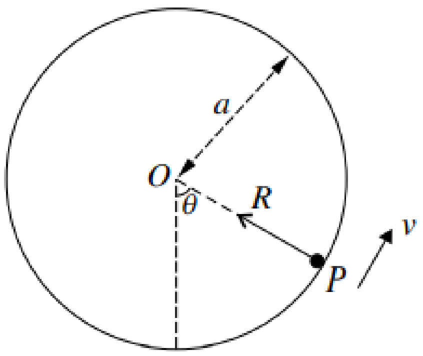


A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point O. With the string taut the particle is travelling in a circular path in a vertical plane. The angle between the string and the downward vertical is $\theta°$ (see diagram). When $\theta = 0$ the speed of P is 7 m s^{-1}.

(i) At the instant when the string is horizontal, find the speed of P and the tension in the string. [4]

(ii) At the instant when the string becomes slack, find the value of θ. [8]

Q2, (Jun 2009, Q7)



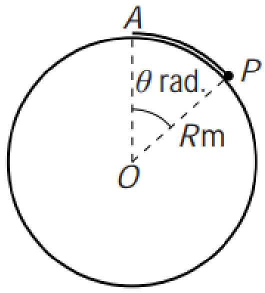
A hollow cylinder has internal radius a. The cylinder is fixed with its axis horizontal. A particle P of mass m is at rest in contact with the smooth inner surface of the cylinder. P is given a horizontal velocity u, in a vertical plane perpendicular to the axis of the cylinder, and begins to move in a vertical circle. While P remains in contact with the surface, OP makes an angle θ with the downward vertical, where O is the centre of the circle. The speed of P is v and the magnitude of the force exerted on P by the surface is R (see diagram).

(i) Find v^2 in terms of u, a, g and θ and show that $R = \dfrac{mu^2}{a} + mg(3\cos\theta - 2)$. **[7]**

(ii) Given that P just reaches the highest point of the circle, find u^2 in terms of a and g, and show that in this case the least value of v^2 is ag. **[4]**

(iii) Given instead that P oscillates between $\theta = \pm\frac{1}{6}\pi$ radians, find u^2 in terms of a and g. **[2]**

Q3, (Jan 2012, Q7)



One end of a light elastic string, of natural length $\frac{2}{3}R$ m and with modulus of elasticity $1.2mg$ N, is attached to the highest point A of a smooth fixed sphere with centre O and radius R m. A particle P of mass m kg is attached to the other end of the string and is in contact with the surface of the sphere, where the angle AOP is equal to θ radians (see diagram).

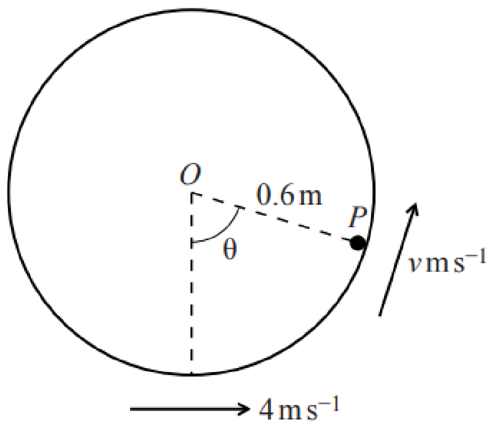
(i) Given that P is in equilibrium at the point where $\theta = \alpha$, show that $1.8\alpha - \sin\alpha - 1.2 = 0$. Hence show that $\alpha = 1.18$ correct to 3 significant figures. **[7]**

P is now released from rest at the point of the surface of the sphere where $\theta = \frac{2}{3}$, and starts to move downwards on the surface. For an instant when $\theta = \alpha$,

(ii) state the direction of the acceleration of P, **[1]**

(iii) find the magnitude of the acceleration of P. **[7]**

Q4, (Jun 2012, Q4)

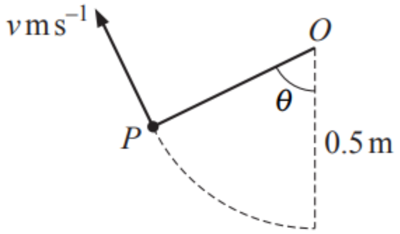


A hollow cylinder is fixed with its axis horizontal. The inner surface of the cylinder is smooth and has radius 0.6 m. A particle P of mass 0.45 kg is projected horizontally with speed 4 m s^{-1} from the lowest point of a vertical cross-section of the cylinder and moves in the plane of the cross-section, which is perpendicular to the axis of the cylinder. While P remains in contact with the surface, its speed is v m s^{-1} when OP makes an angle θ with the downward vertical at O, where O is the centre of the cross-section (see diagram). The force exerted on P by the surface is R N.

(i) Show that $v^2 = 4.24 + 11.76\cos\theta$ and find an expression for R in terms of θ. **[6]**

(ii) Find the speed of P at the instant when it leaves the surface. **[4]**

Q5, (Jun 2015, Q7)



One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. P is projected horizontally from the point 0.5 m below O with speed u m s^{-1}. When the string makes an angle of θ with the downward vertical the particle has speed v m s^{-1} (see diagram).

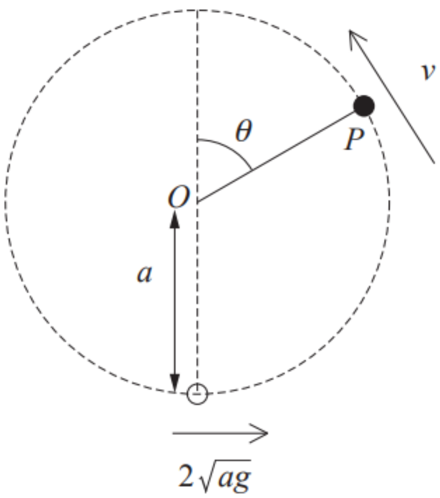
(i) Show that, while the string is taut, the tension, T N, in the string is given by

$$T = 5.88\cos\theta + 0.4u^2 - 3.92.$$ **[5]**

(ii) Find the least value of u for which the particle will move in a complete circle. **[3]**

(iii) If in fact $u = 3.5$ m s^{-1}, find the speed of the particle at the point where the string first becomes slack. **[4]**

Q6, (Jun 2016, Q5)



One end of a light inextensible string of length a is attached to a fixed point O. A particle P of mass m is attached to the other end of the string and hangs at rest. P is then projected horizontally from this position with speed $2\sqrt{ag}$. When the string makes an angle θ with the upward vertical P has speed v (see diagram). The tension in the string is T.

(i) Find an expression for T in terms of m, g and θ, and hence find the height of P above its initial level when the string becomes slack. **[6]**

P is now projected horizontally from the same initial position with speed U.

(ii) Find the set of values of U for which the string does not remain taut in the subsequent motion. **[5]**