**Dimensional Analysis Exam Questions (From OCR 4763)**

**Q1, (Jan 2006, Q1a)**

**(i)** Write down the dimensions of force. [1]

The period, $t$, of a vibrating wire depends on its tension, $F$, its length, $l$, and its mass per unit length, $\sigma$.

**(ii)** Assuming that the relationship is of the form $t = kF^{\alpha}l^{\beta}\sigma^{\gamma}$, where $k$ is a dimensionless constant, use dimensional analysis to determine the values of $\alpha$, $\beta$ and $\gamma$. [6]

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**Q2, (Jun 2006, Q1a)**

**(i)** Find the dimensions of power. [3]

In a particle accelerator operating at power $P$, a charged sphere of radius $r$ and density $\rho$ has its speed increased from $u$ to $2u$ over a distance $x$. A student derives the formula

$$x = \frac{28\pi r^3 u^2 \rho}{9P}.$$

**(ii)** Show that this formula is not dimensionally consistent. [5]

**(iii)** Given that there is only one error in this formula for $x$, obtain the correct formula. [3]

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**Q3, (Jun 2007, Q1a)**

**(i)** Write down the dimensions of the following quantities.

Velocity
Acceleration
Force
Density (which is mass per unit volume)
Pressure (which is force per unit area) [5]

For a fluid with constant density $\rho$, the velocity $v$, pressure $P$ and height $h$ at points on a streamline are related by Bernoulli's equation

$$P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant},$$

where $g$ is the acceleration due to gravity.

**(ii)** Show that the left-hand side of Bernoulli's equation is dimensionally consistent. [4]

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**Q4, (Jan 2007, Q1)**

**(i)** Write down the dimensions of velocity, acceleration and force. [3]

The force $F$ of gravitational attraction between two objects with masses $m_1$ and $m_2$, at a distance $r$ apart, is given by

$$F = \frac{Gm_1m_2}{r^2}$$

where $G$ is the universal constant of gravitation.

**(ii)** Show that the dimensions of $G$ are $\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}$. [2]

**(iii)** In SI units (based on the kilogram, metre and second) the value of $G$ is $6.67 \times 10^{-11}$.

Find the value of $G$ in imperial units based on the pound (0.4536 kg), foot (0.3048 m) and second. [3]

**(iv)** For a planet of mass $m$ and radius $r$, the escape velocity $v$ from the planet's surface is given by

$$v = \sqrt{\frac{2Gm}{r}}.$$

Show that this formula is dimensionally consistent. [3]

**(v)** For a planet in circular orbit of radius $R$ round a star of mass $M$, the time $t$ taken to complete one orbit is given by

$$t = kG^{\alpha}M^{\beta}R^{\gamma}$$

where $k$ is a dimensionless constant.

Use dimensional analysis to find $\alpha$, $\beta$ and $\gamma$. [5]

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**Q5, (Jun 2009, Q3a)**

**(i)** Write down the dimensions of velocity, force and density (which is mass per unit volume). **[3]**

A vehicle moving with velocity $v$ experiences a force $F$, due to air resistance, given by

$$F = \tfrac{1}{2}C\rho^{\alpha}v^{\beta}A^{\gamma}$$

where $\rho$ is the density of the air, $A$ is the cross-sectional area of the vehicle, and $C$ is a dimensionless quantity called the drag coefficient.

**(ii)** Use dimensional analysis to find $\alpha$, $\beta$ and $\gamma$. **[5]**

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**Q6, (Jun 2008, Q1i-iv)**

**(i)** Write down the dimensions of velocity, acceleration and force. [3]

A ball of mass $m$ is thrown vertically upwards with initial velocity $U$. When the velocity of the ball is $v$, it experiences a force $\lambda v^2$ due to air resistance where $\lambda$ is a constant.

**(ii)** Find the dimensions of $\lambda$. [2]

A formula approximating the greatest height $H$ reached by the ball is

$$H \approx \frac{U^2}{2g} - \frac{\lambda U^4}{4mg^2}$$

where $g$ is the acceleration due to gravity.

**(iii)** Show that this formula is dimensionally consistent. [4]

A better approximation has the form $H \approx \frac{U^2}{2g} - \frac{\lambda U^4}{4mg^2} + \frac{1}{6}\lambda^2 U^\alpha m^\beta g^\gamma$.

**(iv)** Use dimensional analysis to find $\alpha$, $\beta$ and $\gamma$. [5]

**Q7, (Jun 2014, Q1a)**

The speed $v$ of sound in a solid material is given by $v = \sqrt{\frac{E}{\rho}}$, where $E$ is Young's modulus for the material and $\rho$ is its density.

**(i)** Find the dimensions of Young's modulus. **[3]**

The density of steel is $7800\,\mathrm{kg\,m^{-3}}$ and the speed of sound in steel is $6100\,\mathrm{m\,s^{-1}}$.

**(ii)** Find Young's modulus for steel, stating the units in which your answer is measured. **[2]**

A tuning fork has cylindrical prongs of radius $r$ and length $l$. The frequency $f$ at which the tuning fork vibrates is given by $f = kc^\alpha E^\beta \rho^\gamma$, where $c = \frac{l^2}{r}$ and $k$ is a dimensionless constant.

**(iii)** Find $\alpha$, $\beta$ and $\gamma$. **[4]**

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**Q8, (Jun 2015, Q1i-iv)**

**(i)** Give the dimensions of force, work and power. **[3]**

The force due to air resistance acting on a car is given by $\lambda v^2$, where $v$ is the speed and $\lambda$ is a constant for that car.

**(ii)** Find the dimensions of $\lambda$. **[2]**

The power $P$ of the car and its maximum speed $U$ are related by the equation $P = \lambda U^3$.

**(iii)** Show that this equation is dimensionally consistent. **[2]**

The time $t$ taken for the car to accelerate from speed $\frac{1}{3}U$ to speed $\frac{2}{3}U$ is given by $t = km^\alpha P^\beta \lambda^\gamma$, where $m$ is the mass of the car and $k$ is a dimensionless constant.

**(iv)** Find $\alpha$, $\beta$ and $\gamma$. **[4]**

**Q9, (Jun 2016, Q1ai)**

In an investigation, small spheres are dropped into a long column of a viscous liquid and their terminal speeds measured. It is thought that the terminal speed $V$ of a sphere depends on a product of powers of its radius $r$, its weight $mg$ and the viscosity $\eta$ of the liquid, and is given by

$$V = kr^{\alpha}(mg)^{\beta}\eta^{\gamma},$$

where $k$ is a dimensionless constant.

**(i)** Given that the dimensions of viscosity are $\mathrm{ML^{-1}T^{-1}}$ find $\alpha$, $\beta$ and $\gamma$. **[6]**

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**Q10, (Jun 2017, Q2a)**

A moving car experiences a force $F$ due to air resistance. It is known that $F$ depends on a product of powers of its velocity $v$, its cross-sectional area $A$ and the air density $\rho$, and is given by

$$F = \frac{1}{2}C\rho^{\alpha}v^{\beta}A^{\gamma},$$

where $C$ is a dimensionless constant known as the drag coefficient.

**(i)** Write down the dimensions of force and density. **[2]**

**(ii)** Use dimensional analysis to find $\alpha$, $\beta$ and $\gamma$. **[5]**

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