Dimensional Analysis Exam Questions (From OCR 4763)

Q1, (Jan 2006, Q1a)

(a)(i	$\mathrm{M\,L\,T^{-2}}$	B1 **1**	Allow $\mathrm{kg\,m\,s^{-2}}$
(ii)	$(\mathrm{T}) = (\mathrm{M\,L\,T^{-2}})^{\alpha}\,(\mathrm{L})^{\beta}\,(\mathrm{M\,L^{-1}})^{\gamma}$	B1 M1	For $\mathrm{M\,L^{-1}}$
	Powers of M: $\alpha+\gamma=0$ of L: $\alpha+\beta-\gamma=0$ of T: $-2\alpha=1$	M2	For three equations Give M1 for one equation
	$\alpha=-\frac{1}{2},\ \ \beta=1,\ \ \gamma=\frac{1}{2}$	A2 **6**	Give A1 for one correct
(iii)	$kF_1^{\alpha}l_1^{\beta}\sigma^{\gamma}=kF_2^{\alpha}l_2^{\beta}\sigma^{\gamma}$	M1	
	$F_1^{-\frac{1}{2}}l_1=F_2^{-\frac{1}{2}}l_2$	A1	Equation relating F_1, F_2, l_1, l_2
	OR $F^{\alpha}l^{\beta}$ is constant M1 F is proportional to l^2 A1		or equivalent
	$F_2 = 90\times\dfrac{2.0^2}{1.2^2}$	M1	
	$=250\ \ (\mathrm{N})$	A1 **4**	

Q2, (Jun 2006, Q1a)

(a)(i)	[Force] $=\mathrm{M\,L\,T^{-2}}$	B1	or [Energy] $=\mathrm{M\,L^2\,T^{-2}}$
	[Power] = [Force] × [Distance] ÷ [Time] = [Force] $\times\mathrm{L\,T^{-1}}$	M1	or [Energy] $\times\mathrm{T^{-1}}$
	$=\mathrm{M\,L^2\,T^{-3}}$	A1 **3**	
(ii)	$[\mathrm{RHS}]=\dfrac{(\mathrm{L})^3(\mathrm{L\,T^{-1}})^2(\mathrm{M\,L^{-3}})}{\mathrm{M\,L^2\,T^{-3}}}$	B1B1	For $(\mathrm{L\,T^{-1}})^2$ and $(\mathrm{M\,L^{-3}})$
	$=\mathrm{T}$	M1 A1	Simplifying dimensions of RHS
	[LHS] = L so equation is not consistent	E1 **5**	With all working correct (cao) *SR* '... $\mathrm{L}=\frac{28}{9}\pi\,\mathrm{T}$, so inconsistent ' can earn B1B1M1A1E0
(iii)	[RHS] needs to be multiplied by $\mathrm{L\,T^{-1}}$ which are the dimensions of *u*	M1 A1	
	Correct formula is $x=\dfrac{28\pi r^3u^3\rho}{9P}$	A1 cao **3**	RHS must appear correctly
	OR $x=k\,r^{\alpha}\,u^{\beta}\,\rho^{\gamma}\,P^{\delta}$ M1 $\beta=3$ A1 $x=\dfrac{28\pi r^3u^3\rho}{9P}$ A1		Equating powers of one dimension

Q3, (Jun 2007, Q1a)

(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$ [Pressure] $= \mathrm{M\,L^{-1}\,T^{-2}}$	B1 B1 B1 B1 B1 **5**	*(Deduct 1 mark if answers given as* $\mathrm{ms^{-1}}$, $\mathrm{ms^{-2}}$, $\mathrm{kg\,ms^{-2}}$ *etc)*
(ii)	$[P] = \mathrm{M\,L^{-1}\,T^{-2}}$ $[\frac{1}{2}\rho v^2] = (\mathrm{M\,L^{-3}})(\mathrm{L\,T^{-1}})^2$ $= \mathrm{M\,L^{-1}\,T^{-2}}$ $[\rho g h] = (\mathrm{M\,L^{-3}})(\mathrm{L\,T^{-2}})(\mathrm{L}) = \mathrm{M\,L^{-1}\,T^{-2}}$ All 3 terms have the same dimensions	M1 A1 A1 E1 **4**	Finding dimensions of 2nd or 3rd term Allow e.g. 'Equation is dimensionally consistent' following correct work

Q4, (Jan 2007, Q1)

(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*Deduct 1 mark if answers given as* $\mathrm{ms^{-1}}$, $\mathrm{ms^{-2}}$, $\mathrm{kg\,ms^{-2}}$
(ii)	$[G] = \dfrac{[F][r^2]}{[m_1][m_2]} = \dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L^2})}{\mathrm{M^2}}$ $= \mathrm{M^{-1}\,L^3\,T^{-2}}$	M1 E1 **2**	
(iii)	$G = 6.67 \times 10^{-11} \times 0.4536 \times \dfrac{1}{(0.3048)^3}$ $= 1.07 \times 10^{-9} \quad (\mathrm{lb^{-1}\,ft^3\,s^{-2}})$	M1M1 A1 **3**	For $\times 0.4536$ and $\times \dfrac{1}{(0.3048)^3}$ *SC* Give M1 for $6.67 \times 10^{-11} \times \dfrac{1}{0.4536} \times (0.3048)^3$ $(= 4.16 \times 10^{-12})$
(iv)	$[\mathrm{RHS}] = \sqrt{\dfrac{(\mathrm{M^{-1}\,L^3\,T^{-2}})(\mathrm{M})}{\mathrm{L}}}$ $= \sqrt{\mathrm{L^2\,T^{-2}}} = \mathrm{L\,T^{-1}}$ which is the same as [LHS]	M1A1 E1 **3**	
(v)	$\mathrm{T} = (\mathrm{M^{-1}\,L^3\,T^{-2}})^{\alpha}\,\mathrm{M}^{\beta}\,\mathrm{L}^{\gamma}$ Powers of M: $-\alpha + \beta = 0$ of L: $3\alpha + \gamma = 0$ of T: $-2\alpha = 1$ $\alpha = -\frac{1}{2}, \quad \beta = -\frac{1}{2}, \quad \gamma = \frac{3}{2}$	M1 M1 A1 M1 A1 **5**	At least two equations Three correct equations Obtaining at least one of α, β, γ

Q5, (Jun 2009, Q3a)

(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 B1 **3**	*Deduct 1 mark for* $\mathrm{ms^{-1}}$ *etc*
(ii)	$\mathrm{M\,L\,T^{-2}} = (\mathrm{M\,L^{-3}})^{\alpha}(\mathrm{L\,T^{-1}})^{\beta}(\mathrm{L^2})^{\gamma}$ $\alpha = 1$ $\beta = 2$ $-3\alpha + \beta + 2\gamma = 1$ $\gamma = 1$	B1 B1 M1A1 A1 **5**	(ft if equation involves α, β and γ)

Q6, (Jun 2008, Q1i-iv)

(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*(Deduct 1 mark if kg, m, s are consistently used instead of M, L, T)*
(ii)	$[\lambda] = \dfrac{[\,\text{Force}\,]}{[\,v^2\,]} = \dfrac{\mathrm{M\,L\,T^{-2}}}{(\mathrm{L\,T^{-1}})^2}$ $= \mathrm{M\,L^{-1}}$	M1 A1 cao **2**	
(iii)	$\left[\dfrac{U^2}{2g}\right] = \dfrac{(\mathrm{L\,T^{-1}})^2}{\mathrm{L\,T^{-2}}} = \mathrm{L}$ $\left[\dfrac{\lambda U^4}{4mg^2}\right] = \dfrac{(\mathrm{M\,L^{-1}})(\mathrm{L\,T^{-1}})^4}{\mathrm{M}(\mathrm{L\,T^{-2}})^2}$ $= \dfrac{\mathrm{M\,L^3\,T^{-4}}}{\mathrm{M\,L^2\,T^{-4}}} = \mathrm{L}$ [*H*] = L ; all 3 terms have the same dimensions	B1 cao M1 A1 cao E1 **4**	*(Condone constants left in)* *Dependent on B1M1A1*
(iv)	$(\mathrm{M\,L^{-1}})^2(\mathrm{L\,T^{-1}})^{\alpha}\,\mathrm{M}^{\beta}(\mathrm{L\,T^{-2}})^{\gamma} = \mathrm{L}$ $\beta = -2$ $-2 + \alpha + \gamma = 1$ $-\alpha - 2\gamma = 0$ $\alpha = 6$ $\gamma = -3$	B1 cao M1 A1 A1 cao A1 cao **5**	At least one equation in α, γ One equation correct

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Q7, (Jun 2014, Q1a)

(i)	$[\rho]=\text{ML}^{-3}$ $[E]=[\rho v^2]=(\text{ML}^{-3})(\text{LT}^{-1})^2$ Dimensions of Young's modulus are $\text{ML}^{-1}\text{T}^{-2}$	B1 M1 A1 **[3]**	 Obtaining dimensions of E	
(ii)	$E=\rho v^2=7800\times6100^2=2.90\times10^{11}$ (3 sf) Units are $\text{kg}\,\text{m}^{-1}\,\text{s}^{-2}$	B1 B1 **[2]**	 OR $\text{N}\,\text{m}^{-2}$ OR Pa	FT provided all powers are non-zero *No FT if derived units involved*
(iii)	$\text{T}^{-1}=\text{L}^{\alpha}(\text{ML}^{-1}\text{T}^{-2})^{\beta}(\text{ML}^{-3})^{\gamma}$ $\beta=\frac{1}{2}$ $\gamma=-\frac{1}{2}$ $\alpha-\beta-3\gamma=0$ $\alpha=-1$	 B1 B1 M1 A1 **[4]**	 CAO FT $\gamma=-\beta$ Equation from powers of L CAO	 *Provided non-zero*

Q8, (Jun 2015, Q1i-iv)

(i)		$[\text{Force}] = \text{M L T}^{-2}$ $[\text{Work}] = \text{M L}^2\,\text{T}^{-2}$ $[\text{Power}] = \text{M L}^2\,\text{T}^{-3}$	B1 B1 B1 **[3]**		*Deduct one mark if kg, m, s used consistently for M, L, T*
(ii)		$[\lambda] = \left[\dfrac{F}{v^2}\right] = \dfrac{\text{M L T}^{-2}}{(\text{L T}^{-1})^2}$ $= \text{M L}^{-1}$	M1 A1 **[2]**	Obtaining dimensions of λ FT $[\text{Force}] \times \text{L}^{-2}\,\text{T}^2$	M0 if $P = \lambda U^3$ used B2 (BOD) for correct answer with no working
(iii)		$[\lambda U^3] = (\text{M L}^{-1})(\text{L T}^{-1})^3 = \text{M L}^2\,\text{T}^{-3}$ Same as power, so dimensionally consistent	M1 E1 **[2]**	Obtaining dimensions of λU^3 Correctly shown	Must be simplified
(iv)		$\text{T} = \text{M}^{\alpha}(\text{M L}^2\,\text{T}^{-3})^{\beta}(\text{M L}^{-1})^{\gamma}$ $\beta = -\frac{1}{3}$ $\alpha + \beta + \gamma = 0,\ \ 2\beta - \gamma = 0$ $\alpha = 1,\ \ \gamma = -\frac{2}{3}$	B1 M1 A1A1 **[4]**	One equation correct (FT) CAO	Equation from powers of M or L If A0 give SC1 for non-zero values with $\gamma = 2\beta$ OR $\alpha + \beta + \gamma = 0$ *(SC1 will usually imply M1)*

Q9, (Jun 2016, Q1ai)

(i)	Units of weight are MLT^{-2} $LT^{-1} = L^{\alpha}(MLT^{-2})^{\beta}(ML^{-1}T^{-1})^{\gamma}$ Compare powers for at least one dimension $0 = \beta + \gamma$ $1 = \alpha + \beta - \gamma$ $-1 = -2\beta - \gamma$ $\alpha = -1,\ \beta = 1,\ \gamma = -1$	B1 M1 M1 A1 A1 A1 **[6]**	 One equation correct Another equation correct All correct

Q10, (Jun 2017, Q2a)

(i)	Dimensions of force: MLT^{-2} Dimensions of density: ML^{-3}	B1 B1 **[2]**	
(ii)	$MLT^{-2} = (ML^{-3})^{\alpha} (LT^{-1})^{\beta} (L^{2})^{\gamma}$ Compare powers for at least one dimension $1 = \alpha$ $1 = -3\alpha + \beta + 2\gamma$ $-2 = -\beta$ $\alpha = 1, \beta = 2, \gamma = 1$	M1 A1 ft M1 A1 cao A1 cao **[5]**	All parts present, dimensions of at least v or A correct At least two equations correct All correct