Coefficient of Restitution (From OCR 4729)

Q1, (Jan 2007, Q2)
Two smooth spheres $A$ and $B$, of equal radius and of masses 0.2 kg and 0.1 kg respectively, are free to move on a smooth horizontal table. $A$ is moving with speed $4 \text{ m s}^{-1}$ when it collides directly with $B$, which is stationary. The collision is perfectly elastic. Calculate the speed of $A$ after the impact. [4]

Q2, (Jan 2007, Q4)
A small sphere of mass 0.2 kg is projected vertically downwards with speed $21 \text{ m s}^{-1}$ from a point at a height of 40 m above horizontal ground. It hits the ground and rebounds vertically upwards, coming to instantaneous rest at its initial point of projection. Ignoring air resistance, calculate

(i) the coefficient of restitution between the sphere and the ground, [6]
(ii) the magnitude of the impulse which the ground exerts on the sphere. [2]

Q3, (Jun 2005, Q4)

\[ \begin{array}{c}
\text{A} \quad 5 \text{ m s}^{-1} \\
\rightarrow \\
\text{B} \quad 2m \\
\text{C} \quad 3m
\end{array} \]

Three smooth spheres $A$, $B$ and $C$, of equal radius and of masses $m$ kg, $2m$ kg and $3m$ kg respectively, lie in a straight line and are free to move on a smooth horizontal table. Sphere $A$ is moving with speed $5 \text{ m s}^{-1}$ when it collides directly with sphere $B$ which is stationary. As a result of the collision $B$ starts to move with speed $2 \text{ m s}^{-1}$.

(i) Find the coefficient of restitution between $A$ and $B$. [4]
(ii) Find, in terms of $m$, the magnitude of the impulse that $A$ exerts on $B$, and state the direction of this impulse. [2]

Sphere $B$ subsequently collides with sphere $C$ which is stationary. As a result of this impact $B$ and $C$ coalesce.

(iii) Show that there will be another collision. [3]

Q4, (Jun 2006, Q8)
Two uniform smooth spheres, $A$ and $B$, have the same radius. The mass of $A$ is 2 kg and the mass of $B$ is $m$ kg. Sphere $A$ is travelling in a straight line on a smooth horizontal surface, with speed $5 \text{ m s}^{-1}$, when it collides directly with sphere $B$, which is at rest. As a result of the collision, sphere $A$ continues in the same direction with a speed of $2 \text{ m s}^{-1}$.

(i) Find the greatest possible value of $m$. [3]

It is given that $m = 1$.

(ii) Find the coefficient of restitution between $A$ and $B$. [3]

On another occasion $A$ and $B$ are travelling towards each other, each with speed $5 \text{ m s}^{-1}$, when they collide directly.

(iii) Find the kinetic energy lost due to the collision. [8]
**Q5, (Jun 2009, Q6)**

Two uniform spheres, \( A \) and \( B \), have the same radius. The mass of \( A \) is 0.4 kg and the mass of \( B \) is 0.2 kg. The spheres \( A \) and \( B \) are travelling in the same direction in a straight line on a smooth horizontal surface, \( A \) with speed 5 m s\(^{-1}\), and \( B \) with speed \( v \) m s\(^{-1}\), where \( v < 5 \). \( A \) collides directly with \( B \) and the impulse between them has magnitude 0.9 N s. Immediately after the collision, the speed of \( B \) is 6 m s\(^{-1}\).

(i) Calculate \( v \). \([3]\)

\( B \) subsequently collides directly with a stationary sphere \( C \) of mass 0.1 kg and the same radius as \( A \) and \( B \). The coefficient of restitution between \( B \) and \( C \) is 0.6.

(ii) Determine whether there will be a further collision between \( A \) and \( B \). \([10]\)

**Q6, (Jan 2011, Q7)**

Three small smooth spheres \( A \), \( B \) and \( C \) of masses 0.2 kg, 0.7 kg and \( m \) kg respectively are free to move in a straight line on a smooth horizontal table. Initially \( B \) and \( C \) are stationary and \( A \) is moving with velocity 1.8 m s\(^{-1}\) directly towards \( B \). The coefficient of restitution for the collision between \( A \) and \( B \) is \( e \). Immediately after this collision the speed of \( A \) is greater than the speed of \( B \).

(i) Calculate the set of possible values of \( e \). \([9]\)

It is now given that the speed of \( B \) immediately after the collision with \( A \) is 0.75 m s\(^{-1}\). \( B \) continues its motion and strikes \( C \) directly in a perfectly elastic collision. \( B \) has speed 0.25 m s\(^{-1}\) immediately after its collision with \( C \).

(ii) Calculate the two possible values of \( m \). \([6]\)

**Q7, (Jan 2013, Q3)**

A particle \( A \) is released from rest from the top of a smooth plane, which makes an angle of 30° with the horizontal. The particle \( A \) collides 2 s later with a particle \( B \), which is moving up a line of greatest slope of the plane. The coefficient of restitution between the particles is 0.4 and the speed of \( B \) immediately before the collision is 2 m s\(^{-1}\). \( B \) has velocity 1 m s\(^{-1}\) down the plane immediately after the collision. Find

(i) the speed of \( A \) immediately after the collision. \([4]\)

(ii) the distance \( A \) moves up the plane after the collision. \([2]\)

The masses of \( A \) and \( B \) are 0.5 kg and \( m \) kg, respectively.

(iii) Find the value of \( m \). \([3]\)

**Q8, (Jun 2014, Q6)**

Two small spheres \( A \) and \( B \), of masses \( 2m \) kg and \( 3m \) kg respectively, are moving in opposite directions along the same straight line towards each other on a smooth horizontal surface. \( A \) has speed 4 m s\(^{-1}\) and \( B \) has speed 2 m s\(^{-1}\) before they collide. The coefficient of restitution between \( A \) and \( B \) is 0.4.

(i) Find the speed of each sphere after the collision. \([6]\)

(ii) Find, in terms of \( m \), the loss of kinetic energy during the collision. \([4]\)

(iii) Given that the magnitude of the impulse exerted on \( A \) by \( B \) during the collision is 2.52 N s, find \( m \). \([3]\)
Q9, (Jun 2013, Q6)

The masses of two particles $A$ and $B$ are $0.2$ kg and $m$ kg respectively. The particles are moving with constant speeds $4 \text{ m s}^{-1}$ and $u \text{ m s}^{-1}$ in the same horizontal line and in the same direction (see diagram). The two particles collide and the coefficient of restitution between the particles is $e$. After the collision, $A$ and $B$ continue in the same direction with speeds $4(1 - e + e^2) \text{ m s}^{-1}$ and $4 \text{ m s}^{-1}$ respectively.

(i) Find $u$ and $m$ in terms of $e$. [6]

(ii) Find the value of $e$ for which the speed of $A$ after the collision is least and find, in this case, the total loss in kinetic energy due to the collision. [5]

(iii) Find the possible values of $e$ for which the magnitude of the impulse that $B$ exerts on $A$ is $0.192 \text{ Ns}$. [4]

Q10, (Jun 2015, Q5)

A small sphere of mass $0.2$ kg is projected vertically downwards with a speed of $5 \text{ m s}^{-1}$ from a height of $1.6 \text{ m}$ above horizontal ground. It hits the ground and rebounds vertically upwards coming to instantaneous rest at a height of $h$ m above the ground. The coefficient of restitution between the sphere and the ground is $0.7$.

(i) Find $h$. [4]

(ii) Find the magnitude and direction of the impulse exerted on the sphere by the ground. [3]

(iii) Find the loss of energy of the sphere between the instant of projection and the instant it comes to instantaneous rest at height $h$ m. [3]

Q11, (Jun 2016, Q6)

The masses of two particles $A$ and $B$ are $4$ kg and $3$ kg respectively. The particles are moving towards each other along a straight line on a smooth horizontal surface. $A$ has speed $8 \text{ m s}^{-1}$ and $B$ has speed $10 \text{ m s}^{-1}$ before they collide. The kinetic energy lost due to the collision is $121.5$ J.

(i) Find the speed and direction of motion of each particle after the collision. [8]

(ii) Find the coefficient of restitution between $A$ and $B$. [2]