Critical Regions in Hypothesis Tests

The critical region of a hypothesis test (aka the rejection region) is the set of x values which would lead to the null hypothesis being rejected.

A single observation, x, is taken from a binomial distribution $B(10, p)$ and a value of 5 is obtained. Use this observation to test H_0: $p = 0.25$ against H_1: $p > 0.25$ using a 5% significance level.

We will use critical regions to perform this test

We need the set of X values such that $P(X \geqslant k) \leqslant 0.05$ i.e. every single value we can observe that would lead to H_0 being rejected.

We do this by trial and improvement:

k	$P(X \geqslant k)$
10	9.537×10^{-7}
9	2.956×10^{-5}
8	4.158×10^{-4}
7	
6	0.0197
5	0.07

$P(X \geqslant 10) = 1 - P(X \leqslant 9) = 9.537 \times 10^{-7}$ (R)

$P(X \geqslant 9) = 1 - P(X \leqslant 8) = 2.956 \times 10^{-5}$ (R)

$P(X \geqslant 8) = 1 - P(X \leqslant 7) = 4.158 \times 10^{-4}$ (R)

$P(X \geqslant 6) = 1 - P(X \leqslant 5) = 0.0197$ (R)

$P(X \geqslant 5) = 1 - P(X \leqslant 4) = 0.07$ (A)

We have seen crossing over boundary from reject to accept $\therefore$ we can be certain we have found the entire critical region

$\therefore$ Critical region is $X \geqslant 6$

Using 'List Mode' in Binomial CD to speed this up

1. Select 'list' on Binomial CD.
2. Starting at the biggest value (if we are testing the right tail) or the smallest (if we are testing the left tail), type in lots of numbers.

Left Tail

3. Look for $P(X \leq k)$ crossing over sig level boundary
4. Summarise

Right Tail

3. Look for the probability crossing over the (1 − SIG LEVEL) boundary
4. Write down the calculations either side of this boundary

$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9803$

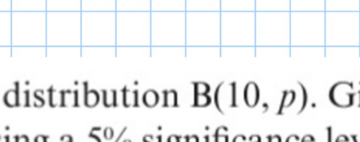
$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9218$

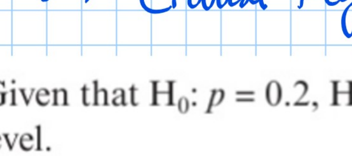
5. Summarise

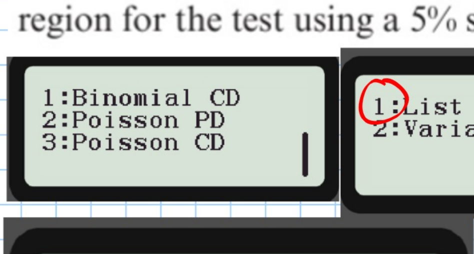
$P(X \geq 6) = 0.0197 < 0.05$
$P(X \geq 5) = 0.0782 > 0.05$

$\therefore$ Critical Region is $X \geq 6$

A test statistic has a distribution B(10, p). Given that H_0: $p = 0.2$, H_1: $p > 0.2$, find the critical region for the test using a 5% significance level.







$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.967 = 0.033 < 0.05$

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.879 = 0.121 > 0.05$

$\therefore$ Critical region is $X \geq 5$

A random variable has a distribution B(20, p). A single observation is used to test H_0: $p = 0.15$ against H_1: $p < 0.15$. Using a 5% level of significance, find the critical region of this test.

Since left tail, we just need to cross the 0·05 boundary

$P(X \leq 0) = 0{\cdot}0387 < 0{\cdot}05$
$P(X \leq 1) = 0{\cdot}1755 > 0{\cdot}05$

$\therefore$ Critical region is $X = 0$ ($X \leq 0$ acceptable)

The test is performed and $X = 3$ is observed. Conclude

$3 > 0$ $\therefore$ Do not reject H_0

A random variable has distribution B(20, p). A single observation is used to test H_0: $p = 0.4$ against H_1: $p \neq 0.4$. Two tail test $\therefore$ two critical regions

a Using the 5% level of significance, find the critical region of this test. **(3 marks)**

b Write down the actual significance level of the test. **(1 mark)**

a/ Two tails $\therefore$ 0·025 sig level each tail

Left: $P(X \leq 3) = 0{\cdot}0160 < 0{\cdot}025$
$P(X \leq 4) = 0{\cdot}0510 > 0{\cdot}025$

$\therefore$ Left critical region is $X \leq 3$

Right: $P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0{\cdot}979 = 0{\cdot}021 < 0{\cdot}025$
$P(X \geq 12)$ $1 - P(X \leq 11) = 1 - 0{\cdot}943 = 0{\cdot}057 > 0{\cdot}025$

$\therefore$ Right critical region is $X \geq 13$

$\therefore$ Critical region is $X \leq 3$ or $X \geq 13$

b/ [Actual sig level is the probability contained within the rejection region]

$P(X \leq 3) + P(X \geq 13) = 0{\cdot}01596 + 1 - 0{\cdot}978797$

$= 0{\cdot}0370$