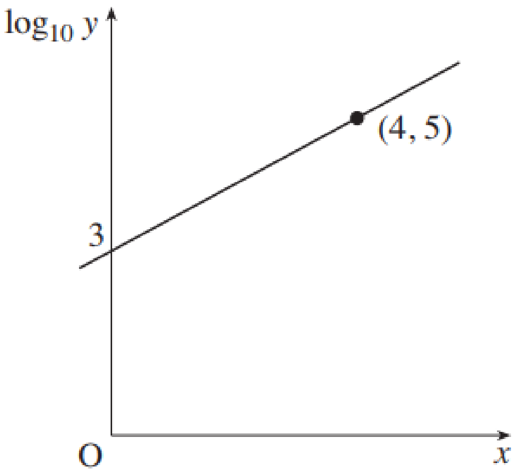
Modelling With Exponentials (From OCR MEI 4752)

Note: All inserts referred to in the question are appended at the end of this document

Q1, (Jan 2006, Q9)



Not to scale

Fig. 9

The graph of $\log_{10} y$ against x is a straight line as shown in Fig. 9.

(i) Find the equation for $\log_{10} y$ in terms of x. [3]

(ii) Find the equation for y in terms of x. [2]

Q2, (Jun 2006, Q12)

Answer the whole of this question on the insert provided.

A colony of bats is increasing. The population, P, is modelled by $P = a \times 10^{bt}$, where t is the time in years after 2000.

(i) Show that, according to this model, the graph of $\log_{10} P$ against t should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis. [3]

(ii) The table gives the data for the population from 2001 to 2005.

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800

Complete the table of values on the insert, and plot $\log_{10} P$ against t. Draw a line of best fit for the data. [3]

(iii) Use your graph to find the equation for P in terms of t. [4]

(iv) Predict the population in 2008 according to this model. [2]

Q3, (Jun 2008, Q13)

The percentage of the adult population visiting the cinema in Great Britain has tended to increase since the 1980s. The table shows the results of surveys in various years.

Year	1986/87	1991/92	1996/97	1999/00	2000/01	2001/02
Percentage of the adult population visiting the cinema	31	44	54	56	55	57

Source: Department of National Statistics, www.statistics.gov.uk

This growth may be modelled by an equation of the form

$$P = at^b,$$

where P is the percentage of the adult population visiting the cinema, t is the number of years after the year 1985/86 and a and b are constants to be determined.

(i) Show that, according to this model, the graph of $\log_{10} P$ against $\log_{10} t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis. [3]

Answer part (ii) of this question on the insert provided.

(ii) Complete the table of values on the insert, and plot $\log_{10} P$ against $\log_{10} t$. Draw by eye a line of best fit for the data. [4]

(iii) Use your graph to find the equation for P in terms of t. [4]

(iv) Predict the percentage of the adult population visiting the cinema in the year 2007/2008 (i.e. when $t = 22$), according to this model. [1]

Q4, (Jun 2012, Q6)

Fig. 6 shows the relationship between $\log_{10} x$ and $\log_{10} y$.

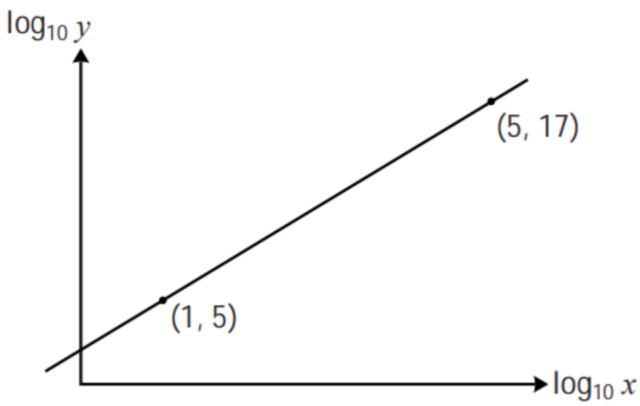


Fig. 6

Find y in terms of x. **[5]**

Q5, (Jun 2009, Q10)

Answer part (i) of this question on the insert provided.

Ash trees grow quickly for the first years of their life, then more slowly. This table shows the height of a tree at various ages.

Age (t years)	4	7	10	15	20	40
Height (h m)	4	9	12	17	19	26

The height, h m, of an ash tree when it is t years old may be modelled by an equation of the form

$$h = a\log_{10} t + b.$$

(i) **On the insert**, complete the table and plot h against $\log_{10} t$, drawing by eye a line of best fit. **[3]**

(ii) Use your graph to find an equation for h in terms of $\log_{10} t$ for this model. **[3]**

(iii) Find the height of the tree at age 100 years, as predicted by this model. **[1]**

(iv) Find the age of the tree when it reaches a height of 29 m, according to this model. **[3]**

(v) Comment on the suitability of the model when the tree is very young. **[2]**

Q6, (Jun 2014, Q13)

The thickness of a glacier has been measured every five years from 1960 to 2010.
The table shows the reduction in thickness from its measurement in 1960.

Year	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010
Number of years since 1960 (t)	5	10	15	20	25	30	35	40	45	50
Reduction in thickness since 1960 (h m)	0.7	1.0	1.7	2.3	3.6	4.7	6.0	8.2	12	15.9

An exponential model may be used for these data, assuming that the relationship between h and t is of the form $h = a \times 10^{bt}$, where a and b are constants to be determined.

(i) Show that this relationship may be expressed in the form $\log_{10} h = mt + c$, stating the values of m and c in terms of a and b. **[2]**

(ii) Complete the table of values in the answer book, giving your answers correct to 2 decimal places, and plot the graph of $\log_{10} h$ against t, drawing by eye a line of best fit. **[4]**

(iii) Use your graph to find h in terms of t for this model. **[4]**

(iv) Calculate by how much the glacier will reduce in thickness between 2010 and 2020, according to the model. **[2]**

(v) Give one reason why this model will not be suitable in the long term. **[1]**

Q7, (Jun 2015, Q8)

Fig. 8 shows the graph of $\log_{10} y$ against $\log_{10} x$. It is a straight line passing through the points $(2, 8)$ and $(0, 2)$.

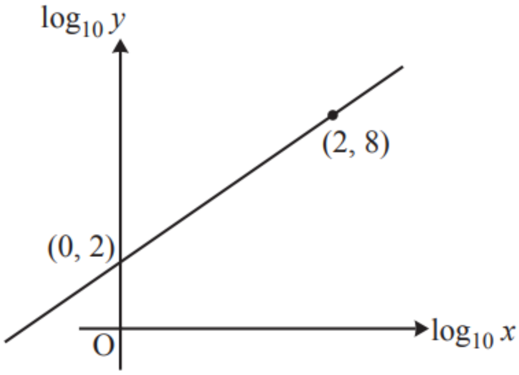


Fig. 8

Find the equation relating $\log_{10} y$ and $\log_{10} x$ and hence find the equation relating y and x. **[4]**

Q8, (Jun 2016, Q11)

There are many different flu viruses. The numbers of flu viruses detected in the first few weeks of the 2012–2013 flu epidemic in the UK were as follows.

Week	1	2	3	4	5	6	7	8	9	10
Number of flu viruses	7	10	24	32	40	38	63	96	234	480

These data may be modelled by an equation of the form $y = a \times 10^{bt}$, where y is the number of flu viruses detected in week t of the epidemic, and a and b are constants to be determined.

(i) Explain why this model leads to a straight-line graph of $\log_{10} y$ against t. State the gradient and intercept of this graph in terms of a and b. **[3]**

(ii) Complete the values of $\log_{10} y$ in the table, draw the graph of $\log_{10} y$ against t, and draw by eye a line of best fit for the data.

Hence determine the values of a and b and the equation for y in terms of t for this model. **[8]**

During the decline of the epidemic, an appropriate model was

$$y = 921 \times 10^{-0.137w},$$

where y is the number of flu viruses detected in week w of the decline.

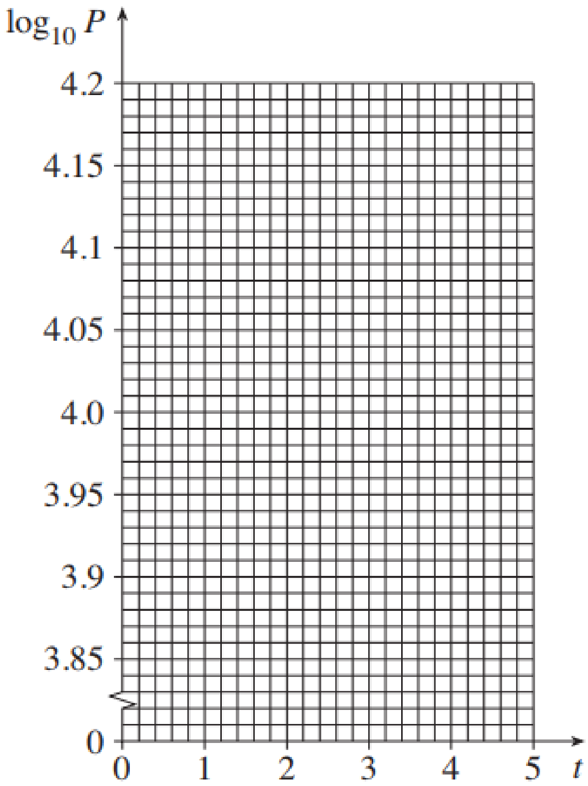
(iii) Use this to find the number of viruses detected in week 4 of the decline. **[1]**

Answer Sheets for Questions

Insert for Q2

(ii)

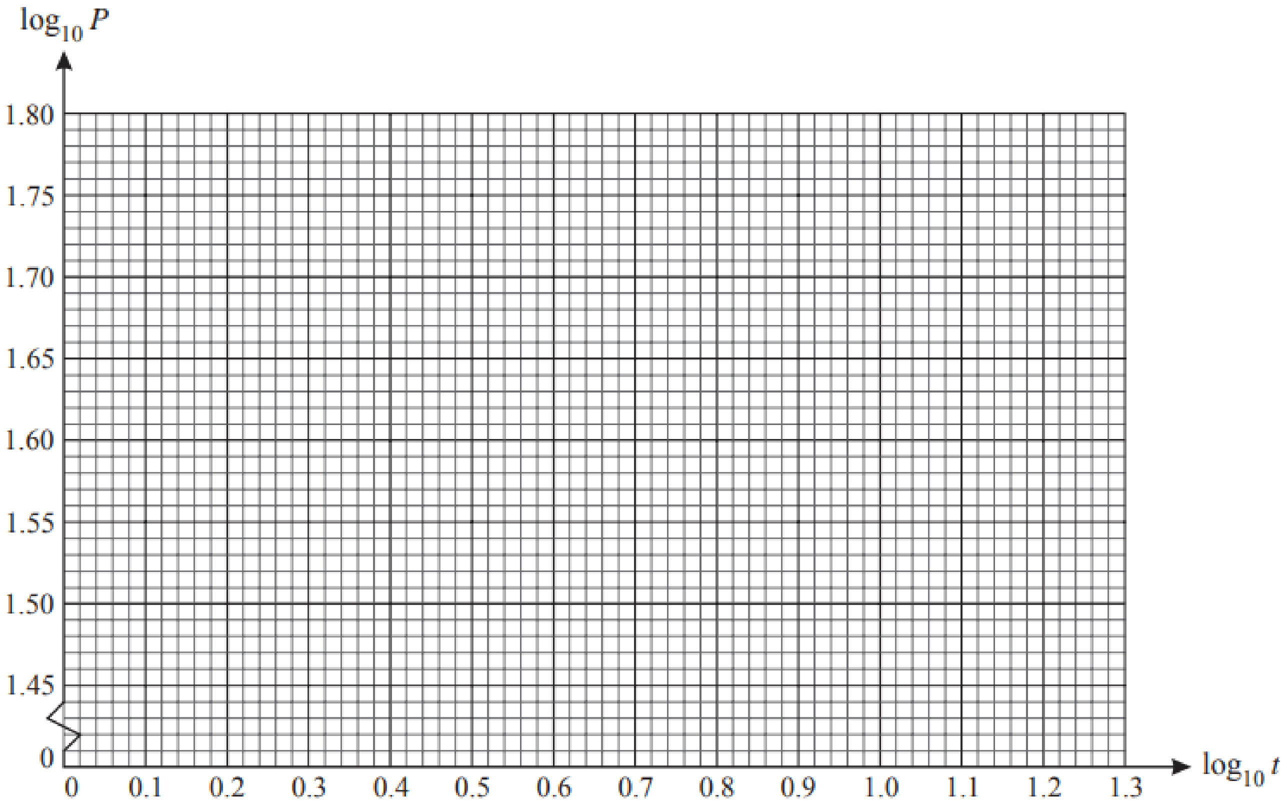
Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800
$\log_{10} P$					



Insert for Q3

(ii)

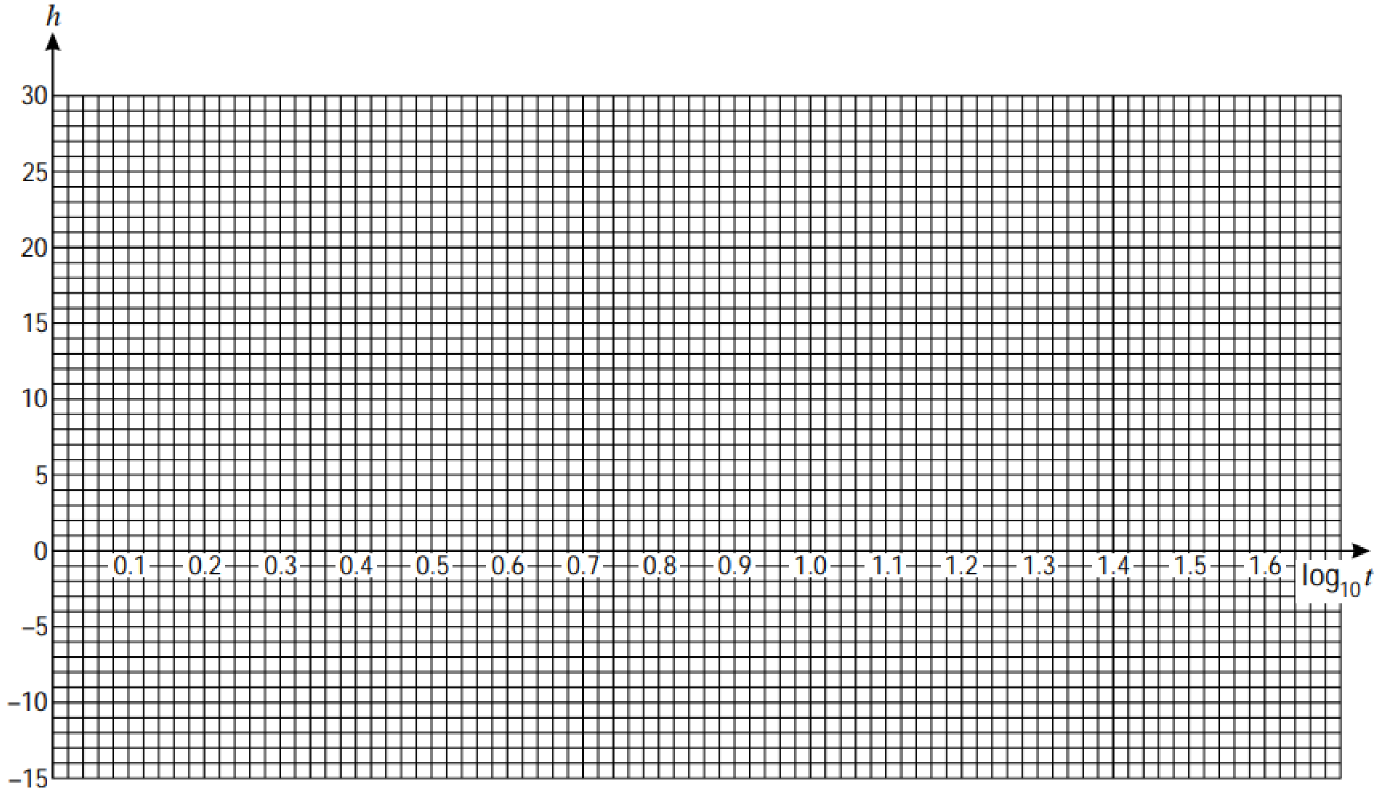
Year	1986/87	1991/92	1996/97	1999/00	2000/01	2001/02
t	1	6	11	14	15	16
P	31	44	54	56	55	57
$\log_{10} t$			1.04			
$\log_{10} P$			1.73			



Insert for Q5

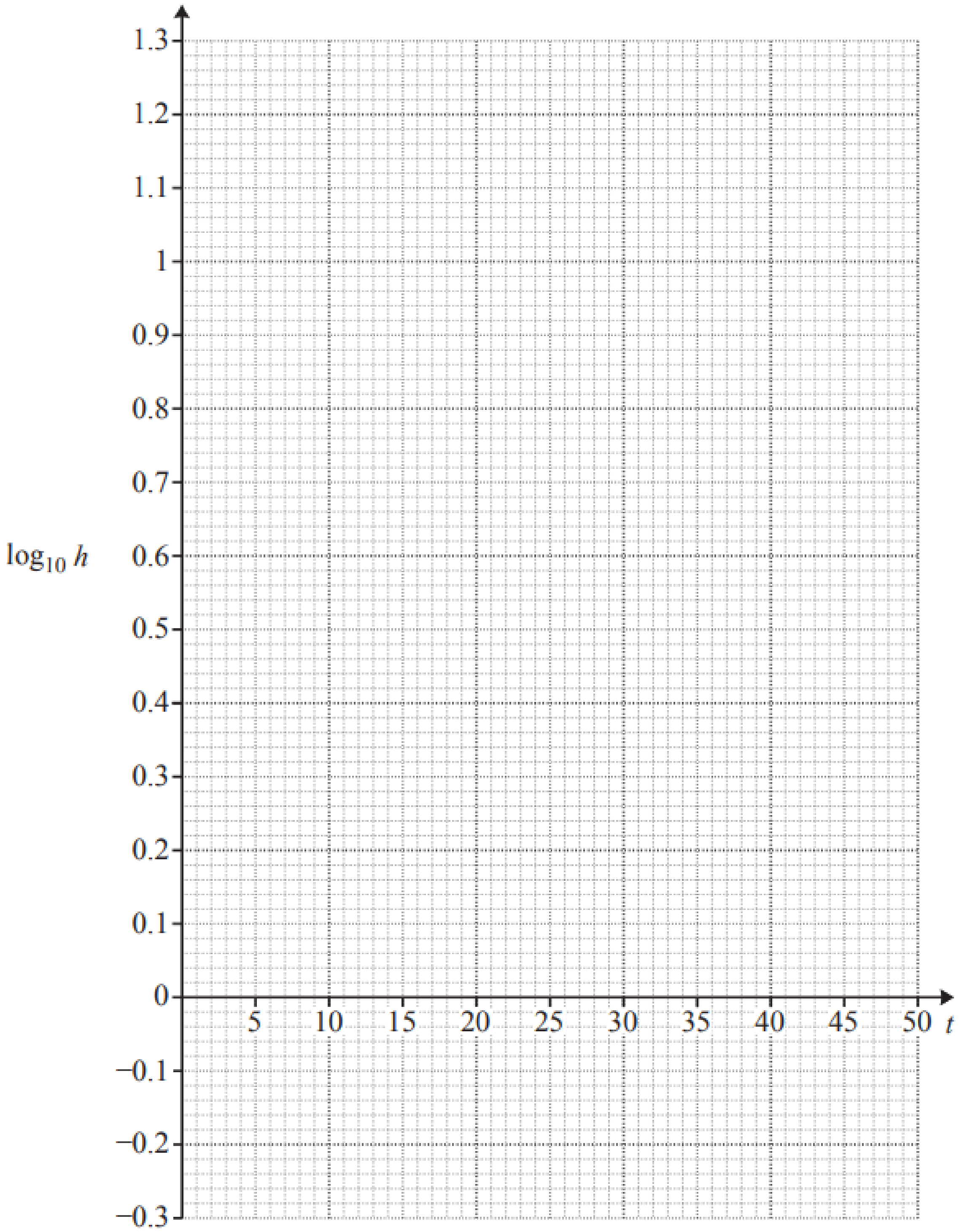
10 (i)

Age (t years)	4	7	10	15	20	40
$\log_{10} t$			1			
Height (h m)	4	9	12	17	19	26



Insert for Q6

t	5	10	15	20	25	30	35	40	45	50
h	0.7	1.0	1.7	2.3	3.6	4.7	6.0	8.2	12	15.9
$\log_{10} h$										



ALevelMathsRevision.com

Insert for Q8

t	1	2	3	4	5	6	7	8	9	10
y	7	10	24	32	40	38	63	96	234	480
$\log_{10}y$	0.85	1	1.38	1.51	1.60