Laws of Logarithms and Logarithmic Equations (From Edexcel 6664)

Q1, (Jun 2006, Q3)

(i) 2	B1	(1)
(ii) $2\log 3 = \log 3^2$ (or $2\log p = \log p^2$)	B1	
$\log_a p + \log_a 11 = \log_a 11p$, $= \log_a 99$ (Allow e.g. $\log_a(3^2 \times 11)$)	M1, A1	(3)
		4
(ii) Ignore 'missing base' or wrong base. The correct answer with no working scores full marks. $\log_a 9 \times \log_a 11 = \log_a 99$, or similar mistakes, score M0 A0.		

Q2, (Jan 2008, Q5)

Method 1 (Substituting a = 3b into second equation at some stage)	
Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$	M1
Substitution of $3b$ for a (or a/3 for b) e.g. $\log_3 3b^2 = 2$	M1
Using base correctly on correctly derived $\log_3 p = q$ e.g. $3b^2 = 3^2$	M1
First correct value $b = \sqrt{3}$ (allow $3^{½}$)	A1
Correct method to find other value (dep. on at least first M mark)	M1
Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1
Method 2 (Working with two equations in $\log_3 a$ and $\log_3 b$)	
" Taking logs" of first equation and " separating" $\log_3 a = \log_3 3 + \log_3 b$ $(= 1 + \log_3 b)$	M1
Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [$\log_3 a = 1½$, $\log_3 b = ½$]	M1
Using base correctly to find a or b	M1
Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1
Correct method for second answer, dep. on first M; correct second answer [Ignore negative values]	M1;A1**[6]**

Q3, (Jan 2009, Q4)

$2\log_5 x = \log_5 (x^2)$, $\quad \log_5(4-x) - \log_5(x^2) = \log_5 \frac{4-x}{x^2}$	B1, M1
$\log\left(\frac{4-x}{x^2}\right) = \log 5 \quad 5x^2 + x - 4 = 0$ **or** $5x^2 + x = 4$ **o.e.**	M1 A1
$(5x-4)(x+1) = 0 \quad x = \frac{4}{5} \quad (x = -1)$	dM1 A1 (6) **[6]**

B1 is awarded for $2\log x = \log x^2$ anywhere.

M1 for correct use of $\log A - \log B = \log \frac{A}{B}$

M1 for replacing 1 by $\log_k k$. **A1** for correct quadratic

($\log(4-x) - \log x^2 = \log 5 \Rightarrow 4 - x - x^2 = 5$ is **B1M0M1A0 M0A0**)

dM1 for attempt to solve quadratic with usual conventions. (Only award if previous two **M** marks have been awarded)

A1 for 4/5 or 0.8 or equivalent (Ignore extra answer).

Q4, (Jun 2009, Q8)

(a)	$\log_2 y = -3 \Rightarrow y = 2^{-3}$	M1	
	$y = \frac{1}{8}$ or 0.125	A1	(2)
(b)	$32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$	M1	
	[or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$]		
	[or $\log_2 32 = \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \frac{\log_{10} 512}{\log_{10} 2}$]		
	$\log_2 32 + \log_2 16 = 9$	A1	
	$(\log x)^2 = \ldots$ or $(\log x)(\log x) = \ldots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2)	M1	
	$\log_2 x = 3 \Rightarrow x = 2^3 = 8$	A1	
	$\log_2 x = -3 \Rightarrow x = 2^{-3} = \frac{1}{8}$	A1ft	(5) **[7]**

Q5, (Jun 2012, Q2)

$2\log x = \log x^2$	B1
$\log_3 x^2 - \log_3(x-2) = \log_3 \dfrac{x^2}{x-2}$	M1
$\dfrac{x^2}{x-2} = 9$	A1 o.e.
Solves $x^2 - 9x + 18 = 0$ to give $x = ...$	M1
$x = 3$, $x = 6$	A1
	Total 5

Q6, (Jan 2010, Q5)

(a)	$\log_x 64 = 2 \quad \Rightarrow \quad 64 = x^2$	M1	
	So $x = 8$	A1	(2)
(b)	$\log_2(11-6x) = \log_2(x-1)^2 + 3$	M1	
	$\log_2\left[\dfrac{11-6x}{(x-1)^2}\right] = 3$	M1	
	$\dfrac{11-6x}{(x-1)^2} = 2^3$	M1	
	$\{11-6x = 8(x^2-2x+1)\}$ and so $0 = 8x^2 - 10x - 3$	A1	
	$0 = (4x+1)(2x-3) \quad \Rightarrow x = ...$	dM1	
	$x = \dfrac{3}{2}, \left[-\dfrac{1}{4}\right]$	A1	(6)
			[8]

Q7, (Jun 2010, Q7)

(a) $2\log_3(x-5) = \log_3(x-5)^2$	B1
$\log_3(x-5)^2 - \log_3(2x-13) = \log_3 \frac{(x-5)^2}{2x-13}$	M1
$\log_3 3 = 1$ seen or used correctly	B1
$\log_3\left(\frac{P}{Q}\right) = 1 \Rightarrow P = 3Q$ $\left\{\frac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13)\right\}$	M1
$x^2 - 16x + 64 = 0$ (*)	A1 cso
	(5)
(b) $(x-8)(x-8) = 0 \Rightarrow x = 8$ Must be seen in part (b). Or: Substitute $x = 8$ into original equation and verify. Having additional solution(s) such as $x = -8$ loses the A mark. $x = 8$ with no working scores both marks.	M1 A1 (2) **7**

Q8, (Jan 2012, Q4)

(a)	$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or $\log y - \log 3 = \log x^2$	B1
	$\log_3 x^2 = 2\log_3 x$	B1
	Using $\log_3 3 = 1$	B1 (3)
(b)	$3x^2 = 28x - 9$	M1
	Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$	M1 A1 (3)
		6

Q9, (Jan 2013, Q6)

(a)	$2\log(x+15)=\log(x+15)^2$		B1
	$\log(x+15)^2-\log x=\log\dfrac{(x+15)^2}{x}$	Correct use of $\log a-\log b=\log\dfrac{a}{b}$	M1
	$2^6=64$ or $\log_2 64=6$	64 used in the correct context	B1
	$\log_2\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=64$	Removes logs correctly	M1
	$\Rightarrow x^2+30x+225=64x$ $or\ x+30+225x^{-1}=64$	Must see expansion of $(x+15)^2$ to score the final mark.	
	$\therefore x^2-34x+225=0$ *		A1
			(5)
(b)	$(x-25)(x-9)=0\Rightarrow x=25$ or $x=9$	M1: Correct attempt to solve the **given** quadratic as far as $x=$... A1: Both 25 and 9	M1 A1
			(2)
			[7]

Q10, Jun 2013, Q7)

7. (i) Method 1	$\log_2\left(\dfrac{2x}{5x+4}\right)=-3$ or $\log_2\left(\dfrac{5x+4}{2x}\right)=3$, or $\log_2\left(\dfrac{5x+4}{x}\right)=4$ (see special case 2)	M1
	$\left(\dfrac{2x}{5x+4}\right)=2^{-3}$ or $\left(\dfrac{5x+4}{2x}\right)=2^3$ or $\left(\dfrac{5x+4}{x}\right)=2^4$ or $\left(\log_2\left(\dfrac{2x}{5x+4}\right)\right)=\log_2\left(\dfrac{1}{8}\right)$	M1
	$16x=5x+4\Rightarrow x=$ (depends on previous Ms and must be this equation or equivalent)	dM1
	$x=\dfrac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work	A1 **cso** **(4)**
7(i) **Method 2**	$\log_2(2x)+3=\log_2(5x+4)$	
	So $\log_2(2x)+\log_2(8)=\log_2(5x+4)$ (3 replaced by $\log_2 8$)	2^{nd} M1
	Then $\log_2(16x)=\log_2(5x+4)$ (addition law of logs)	1^{st} M1
	Then final M1 A1 as before	dM1A1
(ii)	$\log_a y+\log_a 2^3=5$	M1
	$\log_a 8y=5$ — Applies product law of logarithms.	dM1
	$y=\dfrac{1}{8}a^5$ — $y=\dfrac{1}{8}a^5$	A1cao
		(3) **[7]**

Q11, (Jun 2013(R), Q6)

6.(a)	Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$	or Way 2: $\log_3(9x) = \log_3 3^{a+2}$	M1
	$= 2 + a$	$= 2 + a$	A1
			(2)
(b)	Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$	or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$	M1
	$\log x^5 = 5\log x$ or log81=4log3 or log81=4	$= \log_3 3^{5a-4}$	M1
	$= 5a - 4$		A1 cso
			(3)
(c)	$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$		
	Method 1	Method 2	
	$\Rightarrow 2 + a + 5a - 4 = 3$	$\log_3\left(9x.\frac{x^5}{81}\right) = (3 \text{ or } \log 27)$	M1
	$\Rightarrow a = \frac{5}{6}$	$\log_3\left(\frac{x^6}{9}\right) = 3 \text{ or } \log 27$	A1
	$\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a\log_{10} 3$ so $x =$	$\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$	M1
	$x = 2.498$ **or awrt**	$x = 2.498$ **or awrt**	A1
			(4)
	If x = -2.498 appears as well or instead this is A0		**Total 9**